



# Chapter 7

# Experimental Designs for Fitting Response Surfaces — I

## 7.1 Example 7.6, Table 7.6, p. 332

```
#### 7.6
fn.data <- "http://statacumen.com/teach/RSM/data/RSM_EX_07-06.txt"
df.7.6 <- read.table(fn.data, header=TRUE)
df.7.6$block <- factor(df.7.6$block)
str(df.7.6)

## 'data.frame': 14 obs. of 4 variables:
## $ x1 : num -1 -1 1 1 0 0 0 0 0 0 ...
## $ x2 : num -1 1 -1 1 0 0 0 0 0 0 ...
## $ block: Factor w/ 2 levels "1","2": 1 1 1 1 1 1 1 2 2 2 ...
## $ y : num 80.5 81.5 82 83.5 83.9 84.3 84 79.7 79.8 79.5 ...

df.7.6
##      x1      x2 block      y
## 1 -1.000 -1.000     1 80.5
## 2 -1.000  1.000     1 81.5
## 3  1.000 -1.000     1 82.0
## 4  1.000  1.000     1 83.5
## 5  0.000  0.000     1 83.9
## 6  0.000  0.000     1 84.3
## 7  0.000  0.000     1 84.0
## 8  0.000  0.000     2 79.7
## 9  0.000  0.000     2 79.8
## 10 0.000  0.000     2 79.5
## 11  1.414  0.000     2 78.4
## 12 -1.414  0.000     2 75.6
## 13  0.000  1.414     2 78.5
## 14  0.000 -1.414     2 77.0
```

Fit second-order linear model, without blocks.

```
library(rsm)
rsm.7.6.y.S0x1x2 <- rsm(y ~ SO(x1, x2), data = df.7.6)
# externally Studentized residuals
rsm.7.6.y.S0x1x2$studres <- rstudent(rsm.7.6.y.S0x1x2)
summary(rsm.7.6.y.S0x1x2)

##
## Call:
## rsm(formula = y ~ SO(x1, x2), data = df.7.6)
##
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept)  81.866      1.205   67.92 2.5e-12 ***
## x1           0.933      1.044    0.89   0.40
## x2           0.578      1.044    0.55   0.60
## x1:x2        0.125      1.476    0.08   0.93
## x1^2        -1.308      1.087   -1.20   0.26
## x2^2        -0.933      1.087   -0.86   0.42
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Multiple R-squared:  0.283, Adjusted R-squared:  -0.166
```

```
## F-statistic: 0.63 on 5 and 8 DF,  p-value: 0.683
##
## Analysis of Variance Table
##
## Response: y
##           Df Sum Sq Mean Sq F value Pr(>F)
## F0(x1, x2)  2    9.6    4.81    0.55  0.60
## TWI(x1, x2) 1     0.1    0.06    0.01  0.93
## PQ(x1, x2)  2   17.8    8.89    1.02  0.40
## Residuals   8   69.7    8.72
## Lack of fit  3   40.6   13.52    2.32  0.19
## Pure error   5   29.2    5.83
##
## Stationary point of response surface:
##      x1      x2
## 0.3724 0.3345
##
## Eigenanalysis:
## $values
## [1] -0.9229 -1.3183
##
## $vectors
##      [,1] [,2]
## x1 -0.1601 -0.9871
## x2 -0.9871  0.1601
```

Fit second-order linear model, with blocks.

```
library(rsm)
rsm.7.6.y.b.S0x1x2 <- rsm(y ~ block + S0(x1, x2), data = df.7.6)
# externally Studentized residuals
rsm.7.6.y.b.S0x1x2$studres <- rstudent(rsm.7.6.y.b.S0x1x2)
summary(rsm.7.6.y.b.S0x1x2)
##
## Call:
## rsm(formula = y ~ block + S0(x1, x2), data = df.7.6)
##
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept)  84.0954    0.0796 1056.07 < 2e-16 ***
## block2      -4.4575    0.0872  -51.10 2.9e-10 ***
## x1           0.9325    0.0577   16.16 8.4e-07 ***
## x2           0.5777    0.0577   10.01 2.1e-05 ***
## x1:x2        0.1250    0.0816    1.53  0.17
## x1^2        -1.3086    0.0601  -21.79 1.1e-07 ***
## x2^2        -0.9334    0.0601  -15.54 1.1e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Multiple R-squared:  0.998, Adjusted R-squared:  0.996
## F-statistic: 607 on 6 and 7 DF,  p-value: 3.81e-09
##
## Analysis of Variance Table
##
```

```
## Response: y
##           Df Sum Sq Mean Sq F value Pr(>F)
## block      1   69.5    69.5 2611.10 2.9e-10
## F0(x1, x2)  2    9.6     4.8  180.73 9.5e-07
## TWI(x1, x2) 1    0.1     0.1   2.35  0.17
## PQ(x1, x2)  2   17.8     8.9  334.05 1.1e-07
## Residuals   7    0.2     0.0
## Lack of fit  3    0.1     0.0   0.53  0.69
## Pure error  4    0.1     0.0
##
## Stationary point of response surface:
##      x1      x2
## 0.3723 0.3344
##
## Eigenanalysis:
## $values
## [1] -0.9233 -1.3187
##
## $vectors
##      [,1] [,2]
## x1 -0.1601 -0.9871
## x2 -0.9871  0.1601
```

Fit second-order linear model, with blocks (as continuous and coded).

```
df.7.6$block.num <- as.numeric(df.7.6$block) - 1.5
df.7.6
##      x1      x2 block      y block.num
## 1 -1.000 -1.000     1 80.5     -0.5
## 2 -1.000  1.000     1 81.5     -0.5
## 3  1.000 -1.000     1 82.0     -0.5
## 4  1.000  1.000     1 83.5     -0.5
## 5  0.000  0.000     1 83.9     -0.5
## 6  0.000  0.000     1 84.3     -0.5
## 7  0.000  0.000     1 84.0     -0.5
## 8  0.000  0.000     2 79.7      0.5
## 9  0.000  0.000     2 79.8      0.5
## 10 0.000  0.000     2 79.5      0.5
## 11 1.414  0.000     2 78.4      0.5
## 12 -1.414 0.000     2 75.6      0.5
## 13  0.000  1.414     2 78.5      0.5
## 14  0.000 -1.414     2 77.0      0.5

library(rsm)
rsm.7.6.y.bn.S0x1x2 <- rsm(y ~ block.num + S0(x1, x2), data = df.7.6)
# externally Studentized residuals
rsm.7.6.y.bn.S0x1x2$studres <- rstudent(rsm.7.6.y.bn.S0x1x2)
summary(rsm.7.6.y.bn.S0x1x2)

##
## Call:
## rsm(formula = y ~ block.num + S0(x1, x2), data = df.7.6)
##
##           Estimate Std. Error t value Pr(>|t|)
```

```

## (Intercept) 81.8667      0.0666 1228.86 < 2e-16 ***
## block.num   -4.4575      0.0872  -51.10 2.9e-10 ***
## x1          0.9325      0.0577   16.16 8.4e-07 ***
## x2          0.5777      0.0577   10.01 2.1e-05 ***
## x1:x2       0.1250      0.0816    1.53  0.17
## x1^2       -1.3086      0.0601  -21.79 1.1e-07 ***
## x2^2       -0.9334      0.0601  -15.54 1.1e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Multiple R-squared:  0.998, Adjusted R-squared:  0.996
## F-statistic: 607 on 6 and 7 DF, p-value: 3.81e-09
##
## Analysis of Variance Table
##
## Response: y
##           Df Sum Sq Mean Sq F value Pr(>F)
## block.num  1   69.5    69.5 2611.10 2.9e-10
## FO(x1, x2)  2    9.6     4.8  180.73 9.5e-07
## TWI(x1, x2) 1    0.1     0.1   2.35  0.17
## PQ(x1, x2)  2   17.8     8.9  334.05 1.1e-07
## Residuals   7    0.2     0.0
## Lack of fit  3    0.1     0.0   0.53  0.69
## Pure error  4    0.1     0.0
##
## Stationary point of response surface:
##      x1      x2
## 0.3723 0.3344
##
## Eigenanalysis:
## $values
## [1] -0.9233 -1.3187
##
## $vectors
##      [,1] [,2]
## x1 -0.1601 -0.9871
## x2 -0.9871  0.1601

```