Chapter 5

Process Improvement with Steepest Ascent

5.1 Example 5.1, Table 5.1, p. 185

Build a first-order response function. Read data.

```
#### 5.1
fn.data <- "http://statacumen.com/teach/RSM/data/RSM_EX_05-01.txt"
df.5.1 <- read.table(fn.data, header=TRUE)
str(df.5.1)
```

```
## 'data.frame': 8 obs. of  3 variables:
##  $ a: int  -1 1 -1 1 0 0 0 0
##  $ b: int  -1 -1 1 1 0 0 0 0
##  $ y: int  775 670 890 730 745 760 780 720
```

Fit first-order with two-way interaction linear model. This model fit is slightly different than the one in the text; the intercept differs and the significance of the factors.

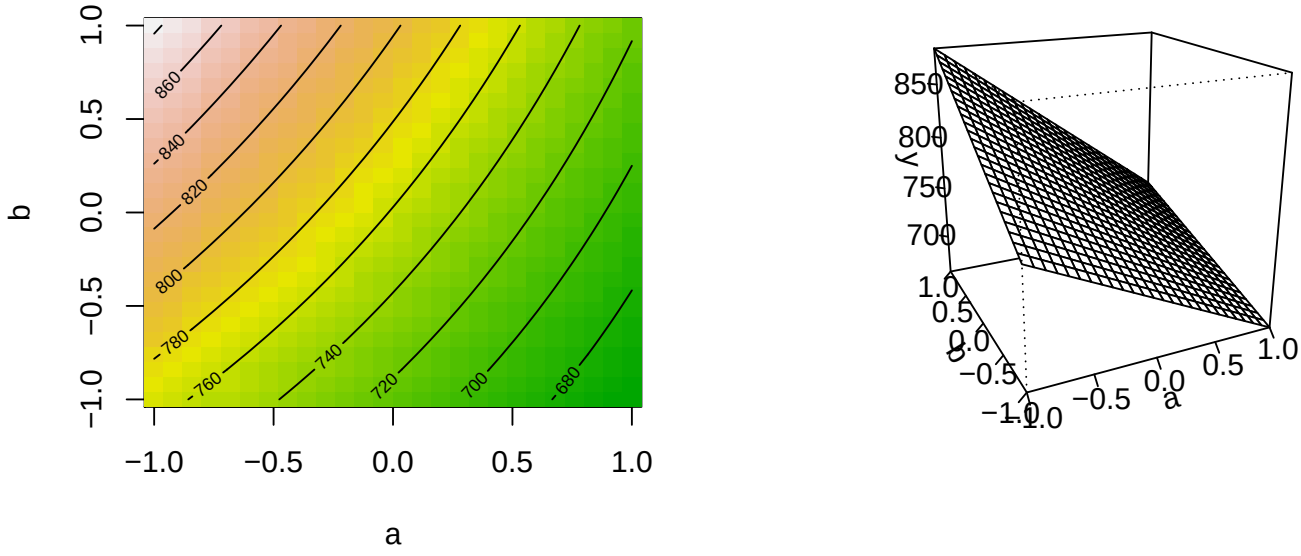
```
library(rsm)
rsm.5.1.y.TWIab <- rsm(y ~ FO(a, b) + TWI(a, b), data = df.5.1)
# externally Studentized residuals
#rsm.5.1.y.TWIab$studres <- rstudent(rsm.5.1.y.TWIab)
summary(rsm.5.1.y.TWIab)
```

```
##
## Call:
## rsm(formula = y ~ FO(a, b) + TWI(a, b), data = df.5.1)
##
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)    758.8        8.6   88.19  9.9e-08 ***
## a              -66.2       12.2   -5.44   0.0055 **
## b               43.8       12.2    3.60   0.0228 *
## a:b            -13.8       12.2   -1.13   0.3216
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Multiple R-squared:  0.916,Adjusted R-squared:  0.854
## F-statistic: 14.6 on 3 and 4 DF,  p-value: 0.0127
##
## Analysis of Variance Table
##
## Response: y
##             Df Sum Sq Mean Sq F value Pr(>F)
## FO(a, b)     2  25212   12606   21.29 0.0074
## TWI(a, b)    1    756     756    1.28 0.3216
## Residuals    4   2369     592
## Lack of fit  1    450     450    0.70 0.4632
## Pure error   3   1919     640
##
## Stationary point of response surface:
##      a      b
##  3.182 -4.818
##
```

```
## Eigenanalysis:
## $values
## [1]  6.875 -6.875
##
## $vectors
##      [,1]    [,2]
## a -0.7071 -0.7071
## b  0.7071 -0.7071
```

Plots indicate the path of steepest ascent is similar to that in Figure 5.3.

```
par(mfrow=c(1,2))
contour(rsm.5.1.y.TWIab, ~ a + b, image = TRUE)
persp(rsm.5.1.y.TWIab, b ~ a, zlab = "y")
```




```
steepest.5.1 <- steepest(rsm.5.1.y.TWIab, dist = seq(0, 7, by = 1))

## Path of steepest ascent from ridge analysis:

steepest.5.1

##   dist      a     b |   yhat
## 1    0  0.000 0.000 |  758.8
## 2    1 -0.805 0.593 |  844.6
## 3    2 -1.573 1.235 |  943.7
## 4    3 -2.321 1.901 | 1056.4
## 5    4 -3.058 2.579 | 1182.6
## 6    5 -3.787 3.265 | 1322.5
## 7    6 -4.511 3.956 | 1476.1
## 8    7 -5.232 4.650 | 1643.3
```

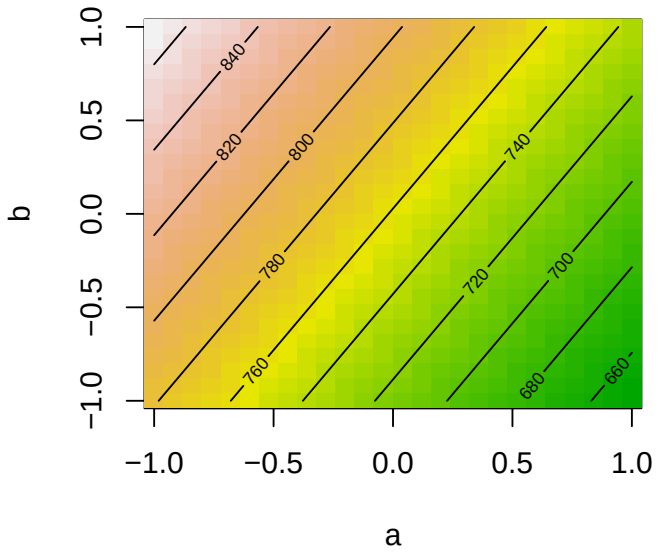
Redo In the text they use a first-order model (equation p. 185) for their steepest ascent calulation.

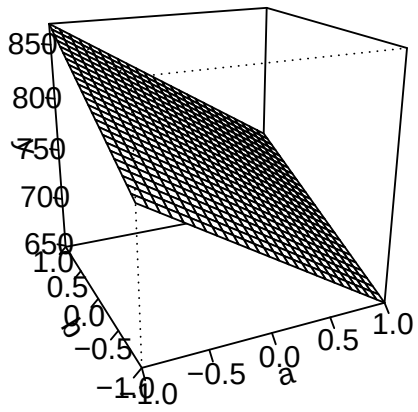
```
library(rsm)
rsm.5.1.y.FOab <- rsm(y ~ FO(a, b), data = df.5.1)
# externally Studentized residuals
#rsm.5.1.y.FOab£studres <- rstudent(rsm.5.1.y.FOab)
summary(rsm.5.1.y.FOab)
```

```
##
## Call:
## rsm(formula = y ~ FO(a, b), data = df.5.1)
##
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   758.75       8.84    85.8  4.1e-09 ***
## a             -66.25      12.50    -5.3   0.0032 **
## b              43.75      12.50     3.5   0.0173 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Multiple R-squared:  0.89,Adjusted R-squared:  0.846
## F-statistic: 20.2 on 2 and 5 DF,  p-value: 0.00404
##
## Analysis of Variance Table
##
## Response: y
##             Df Sum Sq Mean Sq F value Pr(>F)
## FO(a, b)     2  25212   12606   20.17  0.004
## Residuals    5   3125     625
## Lack of fit  2   1206     603    0.94  0.481
## Pure error   3   1919     640
##
## Direction of steepest ascent (at radius 1):
##       a       b
## -0.8345  0.5511
##
## Corresponding increment in original units:
##       a       b
## -0.8345  0.5511
```

Plots indicate the path of steepest ascent is similar to that in Figure 5.3.

```
par(mfrow=c(1,2))
contour(rsm.5.1.y.FOab, ~ a + b, image = TRUE)
persp(rsm.5.1.y.FOab, b ~ a, zlab = "y")
```





This result (going in units of a=1) matches Table 5.3.

```
summary(rsm.5.1.y.FOab)$sa
```

```
##       a       b
## -0.8345  0.5511
```

```
steepest.5.1 <- steepest(rsm.5.1.y.FOab, dist = seq(0, 7, by = 1))
```

```
## Path of steepest ascent from ridge analysis:
```

```
steepest.5.1$a[2]
```

```
## [1] -0.834
```

```
steepest.5.1b <- steepest(rsm.5.1.y.FOab, dist = seq(0, 7, by = 1/abs(steepest.5.1$a[2])))
```

```
## Path of steepest ascent from ridge analysis:
```

```
steepest.5.1b
```

```
##    dist      a     b |   yhat
## 1 0.000  0.000 0.000 |  758.8
## 2 1.199 -1.001 0.661 |  854.0
## 3 2.398 -2.001 1.321 |  949.1
## 4 3.597 -3.002 1.982 | 1044.3
## 5 4.796 -4.002 2.643 | 1139.5
## 6 5.995 -5.003 3.304 | 1234.7
```