Response Surface Methodology, Stat 579 | Homework 08 | Name: ______________________

Fall 2014 | Prof. Erik B. Erhardt

Part I. (50 points) I recommend reading through all the parts of the HW (with my adjustments) before starting; this may save you some work.
MMA-RSM Chapter 8: 8.8 (a),(b),(d), 8.12, 8.14, 8.15.

- For 8.12, (c) refer to 8.15.
- For 8.14, add 7 rather than 6 additional design points. Make your list of candidates a sensible one with all points within the unit cube. Use D-efficiency as the criterion. Is the resulting augmented design a standard design? If so, tell what it is.
- For 8.15, do this just for D-optimality. The formula near the middle of page 387 may be useful to you.

General: Try to do all calculations in R. All R code for the assignment should be included with the part of the problem it addresses (for code and output use a fixed-width font, such as Courier). Code is used to calculate result; text is used to report and interpret results – do not report or interpret results in the code.

(15^{pts}) **1. 8.8**
Consider the following first-order design

$$\mathbf{D} = \begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix}.$$

(a) (5 pts) Is the design first-order orthogonal? Explain?

(b) (5 pts) Give the D-efficiency of this design.

(c) (0 pts) Give the I-efficiency of the design.
Skip (c)

(d) (5 pts) Give the A-efficiency of the design. Hint: The A-efficiency of a design is given by $\frac{\min_{\zeta} \text{tr}\left([\mathbf{M}(\zeta)]^{-1}\right)}{N\text{tr}[(\mathbf{X}^\top\mathbf{X})^{-1}]}$.
Now, of course, for a first-order model with range $[-1, +1]$ on each design variable we have $\min_{\zeta} \text{tr}\left([\mathbf{M}(\zeta)]^{-1}\right) = p$.

(20^{pts}) **2. 8.12** Often a standard second-order design is planned because the practitioner expects that the fitted model will, indeed, be second-order. However, when the analysis is conducted, it is determined that the model is considerably less than order 2. As an example, suppose a $k = 2$ CCD with $a = 1.0$ and two center runs is used and all second-order effects (quadratic and interaction) are very insignificant. The design is used eventually for a first-order model.
For (c) refer to 8.15.
This is a 3^2 design with two center runs, for $n = 10$.

(a) (5 pts) What is the D-efficiency for a $k = 2$ CCD with $a = 1.0$ and two center runs for a first-order model? Comment.

(b) (5 pts) How does the D-efficiency change if there are no center runs?

(c) (5 pts) Is the difference between the results in (a) and (b) expected?

(d) (5 pts) Give the A-efficiency for both designs.

(10^{pts}) **3. 8.14** Consider the following experimental design:

$$\mathbf{D} = \begin{bmatrix} -1 & -1 & -1 \\ 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

A second stage is used, as it becomes obvious that the model is quadratic. Use a computer-generated design package to augment the above design with six more design points in order to accommodate a complete second-order model.
Add 7 rather than 6 additional design points. Make your list of candidates a sensible one with all points within the unit cube. Use D-efficiency as the criterion. Is the resulting augmented design a standard design? If so, tell what it is.

(5^{pts}) **4. 8.15** Consider a first-order orthogonal design with ± 1 levels. Show that the addition of center runs must lower the D-efficiency. Show that the same is true for A-efficiency and for G-efficiency.
Do this just for D-optimality. The formula near the middle of page 387 may be useful to you.