Response Surface Methodology, Stat 579 Homework 07 Name:______________________________
Fall 2014 Prof. Erik B. Erhardt

Part I. (80 points) I recommend reading through all the parts of the HW (with my adjustments) before starting; this may save you some work.
MMA-RSM Chapter 7: 7.1, 7.2, 7.3, 7.5, 7.7, 7.25.

- For 7.7, confirm that the problem statement is correct. If there is an error/are errors, tell what it is/they are and give the correct solution. (There may not be an error.)
- For 7.25, do this for $k = 6(\frac{1}{2}\text{rep})$ instead of $k = 5(\frac{1}{2}\text{rep})$.

General: Try to do all calculations in R. All R code for the assignment should be included with the part of the problem it addresses (for code and output use a fixed-width font, such as Courier). Code is used to calculate result; text is used to report and interpret results – do not report or interpret results in the code.

(15^{pts}) **1. 7.1** Consider the design of an experiment to fit the first-order response model $Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \varepsilon$. The design used is a 2^{5-2} fractional factorial with defining relations $I = ABC = x_1x_2x_3$ and $I = CDE = x_3x_4x_5$. The factors are quantitative, and thus we use the notation x_1, x_2, x_3, ..., rather than A, B, C, ...

(a) (5 pts) Construct the matrix $\mathbf{X}$ for the first-order model using the design described above.
(b) (5 pts) Is the design orthogonal? Explain why or why not.
(c) (5 pts) Is the design a variance-optimal design — that is, one that results in minimum values of $[N\text{Var}(b_i)]/\sigma^2$. Note that the weight N is used in order to take sample size into account.

(25^{pts}) **2. 7.2** Consider the situation of Exercise 7.1. Suppose we use four center runs to augment the 2^{5-2} design constructed.

(a) (5 pts) What is the advantage of using the center runs?
(b) (5 pts) Will the resulting 12-run design be orthogonal? Explain.
(c) (5 pts) Will the resulting design be variance-optimal?
(d) (5 pts) If your answer to (c) is no, give a 12-run first-order design with values of $\text{Var}(b_i)/\sigma^2$ smaller than the 12-run design with center runs.
(e) (5 pts) Give the variances of regression coefficients for both designs — that is, the 2^{5-2}_{III} with four center runs and your design in (d).

(10^{pts}) **3. 7.3** Consider again the design constructed in Exercise 7.1. Suppose the fitted model is first-order and the analysis reveals significant lack of fit and a large effect for the interaction $x_1x_4 = x_2x_5$. As a result, a second phase would suggest an augmentation of the design. Suppose the second phase is a fold-over of the design in Exercise 7.1. Is the resulting design orthogonal and thus variance-optimal for the model

$$y_u = \beta_0 + \sum_{i=1}^{5} \beta_i x_{ui} + \sum_{i<j=2}^{5} \beta_{ij} x_{ui} x_{uj} + \varepsilon_u, \quad u = 1, \ldots, 16.$$

Explain why or why not.

(10^{pts}) **4. 7.5** Consider a situation involving seven design variables when, in fact, a first-order model is required but only eight runs can be used. Design a saturated eight-run design that is variance-optimal.

(10^{pts}) **5. 7.7** Consider the "model inadequacy" material in Section 7.2. The material can be used to nicely illustrate the aliasing ideas discussed in Chapter 4. Aliasing plays an important role when one deals with fractional factorials for fitting first-order and first-order-plus-interaction response surface models. Suppose one fits a first-order model using a 2^{3-1}_{III} with defining relation $I = x_1x_2x_3$.
However, suppose the true model is

$$\text{E}[y] = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3.$$

Using Equation 7.2, we obtain ...
For 7.7, confirm that the problem statement is correct. If there is an error/are errors, tell what it is/they are and give the correct solution. (There may not be an error.)

(10pts) **6. 7.25** Consider Table 7.5. The designs suggested in the table are those that block orthogonally and are rotatable or near-rotatable. For example, consider the design under the column headed $k = 5(\frac{1}{2}\text{rep})$. Construct the design completely, and verify that it meets both conditions for orthogonal blocking.
For 7.25, do this for $k = 6(\frac{1}{2}\text{rep})$ instead of $k = 5(\frac{1}{2}\text{rep})$.