Response Surface Methodology, Stat 579 | Homework 06 | Name: ____________________________
Fall 2014 | | Prof. Erik B. Erhardt

Part I. (80 points) I recommend reading through all the parts of the HW (with my adjustments) before starting; this may save you some work.
MMA-RSM Chapter 6: 6.1, 6.9 (a), (b), (e–g), 6.14 (a), (c).

- For 6.9 (f), constrain the solution to a radius 1.5 of center.
- For 6.14 (c),
  - Use the method I have denoted Method C in class.
  - Take Y1: L=90, T=190; Y2: L=700, T=1300; Y3: L=300, T=500, U=700; Y4: L=50, T=67.5, U=85.
  - Optimize d1: r=0.1; d2: r=0.5; d3: r1=1, r2=0.1; d4: r1=0.2, r2=0.75.
  - Give the value of the desirability function and each response at the optimum. Are the response values within the desired ranges?
  - Use [-2,2] cube for domain of interest.
  - Provide appropriate contour plots.

General: Try to do all calculations in R. All R code for the assignment should be included with the part of the problem it addresses (for code and output use a fixed-width font, such as Courier). Code is used to calculate result; text is used to report and interpret results – do not report or interpret results in the code.

(20^{pts}) **1. 6.1** In a study to determine the nature of a response system that relates dry modulus of rupture (psi) in a certam ceramic material with three important independent vanables, the following quadratic regression equation was determined [see Hackney and Jones (1969)]:

$$\begin{aligned}\hat{y}/100 &= 6.88 - 0.1466x_1^2 + 0.1875x_1x_2 + 0.2050x_1x_3 + 0.0325x_1 \\ &\quad -0.0053x_2^2 - 0.1450x_2x_3 + 0.2588x_2 + 0.1359x_3^2 - 0.1363x_3\end{aligned}$$

The independent variables represent ratios of concentration of various ingredients in the material.

(a) (5 pts) Determine the stationary point.

(b) (5 pts) Put the response surface into canonical form, and determine the nature of the stationary point.

(c) (5 pts) Find the appropriate.expressions relating the canonical variables to the independent variables x_1, x_2. and x_3.

(d) (5 pts) Generate two-dimensional graphs showing contours of constant estimated modulus of rupture. Use $x_3 = -1, 0, 1$.

(30^{pts}) **2. 6.9** 6.9 (a), (b), (e-g)
In Schmidt and Launsby (1990), a simulation program was given that simulates an auto-bumper plating process using thickness as the response with time, temperature, and pH as the design variables. An experiment was conducted in order that a response surface optimization could be accomplished. The coding for the design variables is given by $x_1 = (\text{time}-8)/4$, $x_2 = (\text{temp}-24)/8$, and $x_3 = (\text{nickel}-14)/4$. The design in the natural units is given in Table E6.8 along with the thickness values.
For 6.9 (f), constrain the solution to a radius 1.5 of center.

(a) (5 pts) Write the design in coded form. Name the type of design.

(b) (10 pts) Fit a complete second-order model in the coded metric.

(c) (0 pts) Edit the model by eliminating obviously insignificant terms.
skip (c)

(d) (0 pts) Check model assumptions by plotting residuals against the design variables separately.
skip (d)

(e) (5 pts) The purpose of this experiment was not to merely find optimum conditions (conditions that maximize thickness) but to gain an impression about the role of the three design variables. Show contour plots, fixing levels of nickel at 10, 14, and 18.

In addition, include contour plots through the stationary point.

(f) (5 pts) Use the plots to produce a recommended set of conditions that maximize thickness. For 6.9 (f), constrain the solution to a radius 1.5 of center.

(g) (5 pts) Compute the standard error of prediction at the location of maximum thickness.

(30[pts]) **3. 6.14** (a), (c).

In their paper Derringer and Suich (1980) illustrate their multiple-response procedure with an interesting data set. The central composite design given in Table E6.13 shows data obtained in the development of a tire tread compound on four responses: PICO Abrasion Index, y_1; 200% modulus, y_2; elongation at break, y_3; hardness, y_4. Each column was taken at the 20 sets of conditions shown where x_1, x_2, and x_3 coded levels of the variables x_1 = hydrated silica level, x_2 = silane couplmg agent level, and x_3 = sulfur. The following inequalities represent desirable conditions on the responses: $y_1 > 120$, $y_2 > 1000$, $400 < y_3 < 600$, and $60 < y_4 < 75$.

(a) (10 pts) Fit appropriate response surface models for all responses.

(b) (0 pts) Develop two-dimensional contours (at $x_3 = -1.6, -1, 0, 1, 1.6$) of constant response for all four responses. Can you determine sets of conditions on x_1, x_2, and x_3 that meet the above requirements? If so, list them.
skip (b)

(c) (20 pts) Use the desirability function procedure to determine other competing conditions.
For 6.14 (c),

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