Response Surface Methodology, Stat 579 | Homework 01 | Name:______________________________
Fall 2014 | | Prof. Erik B. Erhardt

Part I. (25 points) Refer to the "Background" reading for Chapter 1.

(5^{pts}) **1.** Refer to (71.) on page 12 of the reading. Assuming the proposed linear model is correct, verify that

$$(\underset{\sim}{y} - \mathbf{X}\underset{\sim}{b})^\top(\underset{\sim}{y} - \mathbf{X}\underset{\sim}{b}) \quad = \quad \underset{\sim}{y}^\top\underset{\sim}{y} - 2\underset{\sim}{b}^\top\mathbf{X}^\top\underset{\sim}{y} + \underset{\sim}{b}^\top\mathbf{X}^\top\mathbf{X}\underset{\sim}{b}.$$

(5^{pts}) **2.** Refer to (73.) on page 12 of the reading. Assuming the proposed linear model is correct, verify

$$\mathrm{E}[\underset{\sim}{b}] \quad = \quad \underset{\sim}{\beta}.$$

(5^{pts}) **3.** Refer to (74.) on page 12 of the reading. Verify

$$\mathrm{E}[(\underset{\sim}{b} - \underset{\sim}{\beta})(\underset{\sim}{b} - \underset{\sim}{\beta})^\top] \quad = \quad \sigma^2(\mathbf{X}^\top\mathbf{X})^{-1}.$$

(10^{pts}) **4.** In (73.) on page 12 of your notes, the claim is that $\mathrm{E}(\underset{\sim}{b}) = \underset{\sim}{\beta}$ if the model true model is $\underset{\sim}{y} = \mathbf{X}\underset{\sim}{\beta} + \underset{\sim}{\varepsilon}$ and $\underset{\sim}{b} = (\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}^\top\underset{\sim}{y}$ is the least squares estimator of $\underset{\sim}{\beta}$. Suppose, however, that the true model contains additional parameters given by vector $\underset{\sim}{\beta}_2$. Therefore, the "true" linear model should be

$$\underset{\sim}{y} \quad = \quad \mathbf{X}\underset{\sim}{\beta} + \mathbf{X}_2\underset{\sim}{\beta}_2 + \underset{\sim}{\varepsilon} \tag{1}$$

where $\mathrm{E}[\underset{\sim}{\varepsilon}] = \underset{\sim}{0}$ and $\mathrm{Var}[\underset{\sim}{\varepsilon}] = \sigma^2\mathbf{I}_n$. Suppose $\mathbf{X}$ is $n \times p$ and $\mathbf{X}_2$ is $n \times p_2$. Show

$$\mathrm{E}[\underset{\sim}{b}] \quad = \quad \underset{\sim}{\beta} + \mathbf{A}\underset{\sim}{\beta}_2,$$

where $\mathbf{A} = (\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}^\top\mathbf{X}_2$.