Chapter 6

A Short Discussion of Observational Studies

Contents

"Thou shall adjust for what thou can not control."

In most scientific studies, the groups being compared do not consist of identical experimental units that have been randomly assigned to receive a treatment. Instead, the groups might be extremely heterogeneous on factors that might be related to a specific response on which you wish to compare the groups. Inferences about the nature of differences among groups in such **observational studies** can be flawed if this heterogeneity is ignored in the statistical analysis.

The following problem emphasizes the care that is needed when analyzing **observational studies**, and highlights the distinction between the **means** and **emmeans** output for a two-way table. The **data are artificial**, but the conclusions are consistent with an interesting analysis conducted by researchers at Sandia National Laboratories.

A representative sample of 550 high school seniors was selected in 1970. A similar sample of 550 was selected in 1990. The final SAT scores (on a 1600 point scale) were obtained for each student[1].

[1] The fake-data example in this chapter is similar to a real-world SAT example illustrated in this

The boxplots for the two samples show heavy-tailed distributions with similar spreads. Given the large sample sizes, the F-test comparing populations is approximately valid even though the population distributions are non-normal.

```
library(tidyverse)

#### Example: SAT
dat_sat <-
  read_csv("http://statacumen.com/teach/ADA2/notes/ADA2_notes_Ch06_sat.csv") %>%
  mutate(
    year = factor(year)
  , eth  = factor(eth)
  )
```

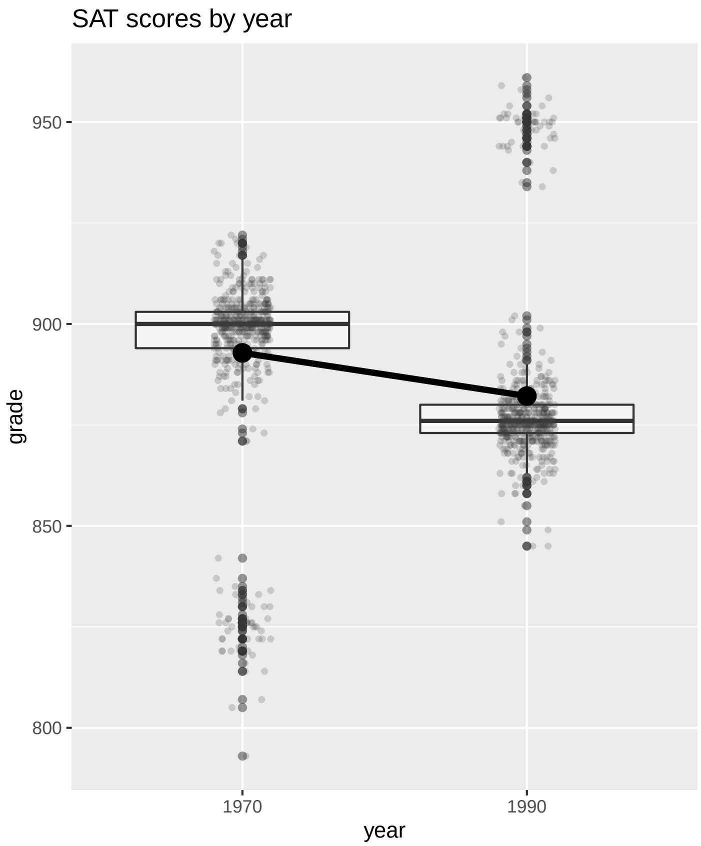
```
## Parsed with column specification:
## cols(
##   year = col_double(),
##   eth = col_double(),
##   grade = col_double()
## )
```

```
# calculate means by year (also calculated below to illustrate emmeans())
sum_sat_mean_y <-
  dat_sat %>%
  group_by(year) %>%
  summarize(
    m = mean(grade)
  )
sum_sat_mean_y
```

```
## # A tibble: 2 x 2
##   year      m
##   <fct> <dbl>
## 1 1970   893.
## 2 1990   882.
```

```
# Interaction plots, ggplot
p <- ggplot(dat_sat, aes(x = year, y = grade))
p <- p + geom_boxplot(alpha = 0.5)
p <- p + geom_point(position = position_jitter(w = 0.1, h = 0), colour="gray25", size=1, alpha = 0.2)
p <- p + geom_point(data = sum_sat_mean_y, aes(y = m), size = 4)
p <- p + geom_line(data = sum_sat_mean_y, aes(y = m, group = 1), size = 1.5)
p <- p + labs(title = "SAT scores by year")
print(p)
```

paper: "Minority Contributions to the SAT Score Turnaround: An Example of Simpson's Paradox" by Howard Wainer, Journal of Educational Statistics, Vol. 11, No. 4 (Winter, 1986), pp. 239–244 http://www.jstor.org/stable/1164696.



A simple analysis might compare the average SAT scores for the two years, to see whether students are scoring higher, lower, or about the same, over time. The one-way **emmeans** and **means** breakdowns of the SAT scores are identical; the average SAT scores for 1970 and 1990 are 892.8 and 882.2, respectively. The one-way ANOVA, combined with the observed averages, indicates that the typical SAT score has decreased significantly (10.7 points) over the 20 year period.

```
lm_g_y <-
  lm(
    grade ~ year
  , data = dat_sat
  , contrasts = list(year = contr.sum)
  )
library(car)
# type III SS
Anova(lm_g_y, type=3)
```

```
## Anova Table (Type III tests)
##
## Response: grade
##                Sum Sq   Df    F value    Pr(>F)
## (Intercept) 866418325    1 1.7076e+06 < 2.2e-16 ***
## year            31410    1 6.1904e+01 8.591e-15 ***
## Residuals      557117 1098
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# balanced with respect to year, so means and emmeans match
sum_sat_mean_y <-
  dat_sat %>%
  group_by(year) %>%
  summarize(
    m = mean(grade)
  )
sum_sat_mean_y
```

```
## # A tibble: 2 x 2
##   year      m
##   <fct> <dbl>
## 1 1970   893.
## 2 1990   882.
```

```
cont_y <-
  emmeans::emmeans(
    lm_g_y
  , specs = "year"
  )
cont_y
```

```
##  year emmean     SE   df lower.CL upper.CL
##  1970  892.8 0.9605 1098    891.0    894.7
##  1990  882.2 0.9605 1098    880.3    884.0
##
## Confidence level used: 0.95
```

Should we be alarmed? Should we be concerned that students entering college have fewer skills than students 20 years ago? Should we be pumping billions of dollars into the bloated bureaucracies of our public school systems with the hope that a few of these dollars might be put to good use in programs to enhance performance? This is the consensus among some people in the know, all of whom wax eloquently about the impending inability of the U.S. to compete in the new global economy.

The SAT study is not a well-designed experiment, where a scientist has controlled all the factors that might affect the response (the SAT score) other than the treatment (the year). Even without these controls, there is no randomization of treatments to students selected from a target population.

The SAT study is an **observational study** of two distinct populations.

The observed differences in SAT scores may indeed be due to a decrease in performance. The differences might also be due to factors that make the two populations incomparable for assessing changes in performance over time.

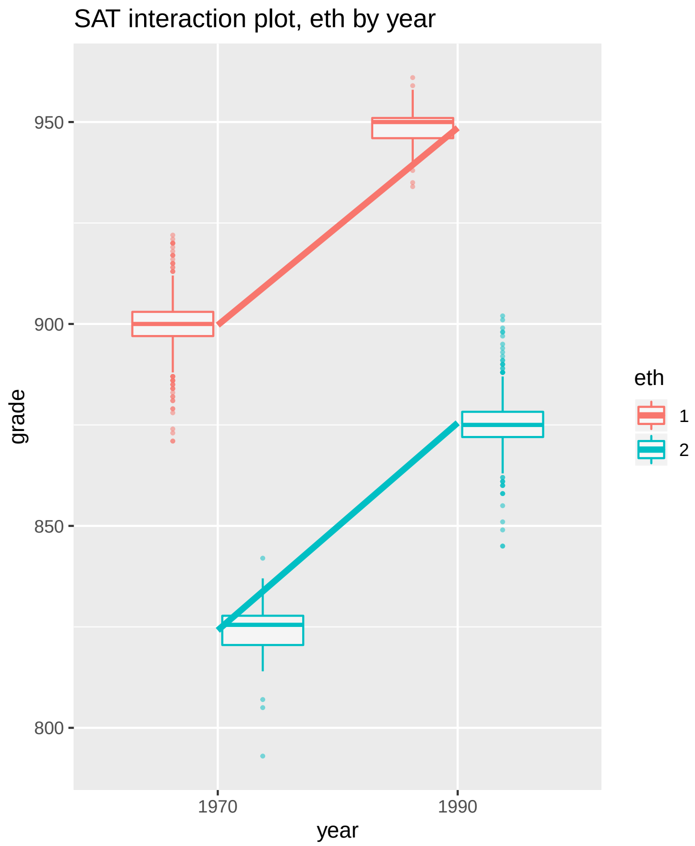
My hypothetical populations have students from two ethnic groups (1 and 2). If you construct box-plots of the SAT scores for the four combinations of ethnicity and year, you see that the typical SAT score **within** each ethnic group has **increased** over time, whereas the typical SAT score **ignoring** ethnicity decreased over time. Is this a paradox, and what are appropriate conclusions in the analysis?

```
sum_sat_mean_ye <-
  dat_sat %>%
  group_by(year, eth) %>%
  summarize(
    m = mean(grade)
  )
sum_sat_mean_ye

## # A tibble: 4 x 3
## # Groups:   year [2]
##   year  eth       m
##   <fct> <fct> <dbl>
## 1 1970  1      900.
## 2 1970  2      824.
## 3 1990  1      949.
## 4 1990  2      876.

# Interaction plots, ggplot
library(ggplot2)
p <- ggplot(dat_sat, aes(x = year, y = grade, colour = eth, shape = eth))
p <- p + geom_boxplot(alpha = 0.5, outlier.size=0.5)
#p <- p + geom_point(data = sum_sat_mean_ye, aes(y = m), size = 4)
p <- p + geom_line(data = sum_sat_mean_ye, aes(y = m, group = eth), size = 1.5)
p <- p + labs(title = "SAT interaction plot, eth by year")
print(p)

#p <- ggplot(dat_sat, aes(x = eth, y = grade, colour = year, shape = year))
#p <- p + geom_boxplot(alpha = 0.5, outlier.size=0.5)
#p <- p + geom_point(data = sum_sat_mean_ye, aes(y = m), size = 4)
#p <- p + geom_line(data = sum_sat_mean_ye, aes(y = m, group = year), size = 1.5)
#p <- p + labs(title = "SAT interaction plot, year by eth")
#print(p)
```



I fit a two-factor model with year and ethnicity effects plus an interaction. The two-factor model gives a method to compare the SAT scores over time, after **adjusting** for the effect of ethnicity on performance. The F-test for comparing years adjusts for ethnicity because it is based on comparing the average SAT scores across years after averaging the cell means over ethnicities, thereby eliminating from the comparison of years any effects due to changes in the ethnic composition of the populations. The two-way analysis is preferable to the unadjusted one-way analysis which **ignores** ethnicity.

```
lm_g_y_e_ye <-
  lm(
    grade ~ year * eth
  , data = dat_sat
  , contrasts = list(year = contr.sum, eth = contr.sum)
  )
## CRITICAL!!!  Unbalanced design warning.
##    The contrast statement above must be included identifying
##    each main effect with "contr.sum" in order for the correct
##    Type III SS to be computed.
##    See http://goanna.cs.rmit.edu.au/~fscholer/anova.php
library(car)
# type III SS
```

```
Anova(lm_g_y_e_ye, type=3)
## Anova Table (Type III tests)
##
## Response: grade
##                Sum Sq   Df    F value  Pr(>F)
## (Intercept) 286085884    1 5.7022e+06 < 2e-16 ***
## year           228283    1 4.5501e+03 < 2e-16 ***
## eth            501984    1 1.0005e+04 < 2e-16 ***
## year:eth          145    1 2.8904e+00 0.08939 .
## Residuals       54988 1096
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The year and ethnicity main effects are significant in the two factor model, but the interaction is not. The marginal **emmeans** indicate that the average SAT score increased significantly over time when averaged over ethnicities. This is consistent with the cell mean SAT scores increasing over time **within** each ethnic group. Given the lack of a significant interaction, the expected increase in SAT scores from 1970 to 1990 **within each** ethnic group is the difference in marginal averages: 912.0 - 861.9 = 50.1.

```
# unbalanced, don't match (emmeans is correct)
sum_sat_mean_y <-
  dat_sat %>%
  group_by(year) %>%
  summarize(
    m = mean(grade)
  )
sum_sat_mean_y
## # A tibble: 2 x 2
##   year      m
##   <fct> <dbl>
## 1 1970   893.
## 2 1990   882.
cont_ye <-
  emmeans::emmeans(
    lm_g_y_e_ye
  , specs = "year"
  )
## NOTE: Results may be misleading due to involvement in interactions
cont_ye
##  year emmean     SE   df lower.CL upper.CL
##  1970  861.9 0.5253 1096    860.9    863.0
##  1990  912.0 0.5253 1096    911.0    913.1
```

```
##
## Results are averaged over the levels of: eth
## Confidence level used: 0.95
```

```
# unbalanced, don't match (emmeans is correct)
sum_sat_mean_e <-
  dat_sat %>%
  group_by(eth) %>%
  summarize(
    m = mean(grade)
  )
sum_sat_mean_e
```

```
## # A tibble: 2 x 2
##   eth       m
##   <fct> <dbl>
## 1 1      904.
## 2 2      871.
```

```
cont_e <-
  emmeans::emmeans(
    lm_g_y_e_ye
  , specs = "eth"
  )
## NOTE: Results may be misleading due to involvement in interactions
cont_e
```

```
##  eth emmean     SE   df lower.CL upper.CL
##  1    924.1 0.5253 1096    923.1    925.2
##  2    849.8 0.5253 1096    848.8    850.9
##
## Results are averaged over the levels of: year
## Confidence level used: 0.95
```

```
# unbalanced, but highest-order interaction cell means will match
sum_sat_mean_ey <-
  dat_sat %>%
  group_by(eth, year) %>%
  summarize(
    m = mean(grade)
  )
sum_sat_mean_ey
```

```
## # A tibble: 4 x 3
## # Groups:   eth [2]
##   eth   year      m
##   <fct> <fct> <dbl>
## 1 1     1970   900.
## 2 1     1990   949.
## 3 2     1970   824.
## 4 2     1990   876.
```

```
cont_ey <-
```

```
  emmeans::emmeans(
    lm_g_y_e_ye
  , specs = "year"
  , by = "eth"
  )
cont_ey
```

```
## eth = 1:
##  year emmean     SE   df lower.CL upper.CL
##  1970  899.7 0.3168 1096    899.1    900.3
##  1990  948.6 1.0017 1096    946.6    950.5
##
## eth = 2:
##  year emmean     SE   df lower.CL upper.CL
##  1970  824.1 1.0017 1096    822.2    826.1
##  1990  875.5 0.3168 1096    874.9    876.1
##
## Confidence level used: 0.95
```

As noted in the insulin analysis (Sec 5.4.1), the marginal **emmeans** and **means** are different for unbalanced two-factor analyses. The marginal **means** **ignore** the levels of the other factors when averaging responses. The marginal **emmeans** are averages of cell means over the levels of the other factor. Thus, for example, the 1970 **mean** SAT score of 892.8 is the average of the 550 scores selected that year. The 1970 **emmeans** SAT score of 861.9 is midway between the average 1970 SAT scores for the two ethnic groups: $861.9 = (899.7 + 824.1)/2$. Hopefully, this discussion also clarifies why the year marginal **means** are identical in the one and two-factor analyses, but the year **emmeans** are not.

The 1970 and 1990 marginal **means** estimate the typical SAT score ignoring all factors that may influence performance. These marginal averages are not relevant for **understanding** any trends in performance over time because they do not account for changes in the composition of the population that may be related to performance.

The average SAT scores (ignoring ethnicity) decreased from 1970 to 1990 because the ethnic composition of the student population changed. Ten out of every eleven students sampled in 1970 were from the first ethnic group. Only one out of eleven students sampled in 1990 was from this group. Students in the second ethnic group are underachievers, but they are becoming a larger portion of the population over time. The decrease in average (**means**) performance inferred from comparing 1970 to 1990 is **confounded** with the increased representation of the underachievers over time. Once ethnicity was taken into consideration, the typical SAT scores were shown to have increased, rather than decreased.

In summary, the one-way analysis ignoring ethnicity is valid, and allows you to conclude that the typical SAT score has decreased over time, but it does not provide any insight into the nature of the changes that have occurred. A two-factor analysis backed up with a comparison of the marginal **emmeans** is needed to compare performances over time, adjusting for the changes in ethnic composition.

The Sandia study reached the same conclusion. The Sandia team showed that the widely reported decreases in SAT scores over time are due to changes in the ethnic distribution of the student population over time, with individuals in historically underachieving ethnic groups becoming a larger portion of the student population over time.

A more complete analysis of the SAT study would adjust the SAT scores to account for other potential confounding factors, such as sex, and differences due to the number of times the exam was taken. These confounding effects are taken into consideration by including them as effects in the model.

The interpretation of the results from an observational study with several effects of interest, and several confounding variables, is greatly simplified by eliminating the insignificant effects from the model. For example, the year by ethnicity interaction in the SAT study might be omitted from the model to simplify interpretation. The year effects would then be estimated after fitting a two-way additive model with year and ethnicity effects only. The same approach is sometimes used with designed experiments, say the insulin study that we analyzed earlier.

An important caveat The ideas that we discussed on the design and analysis of experiments and observational studies are universal. They apply regardless of whether you are analyzing categorical data, counts, or measurements.