Response Surface Analysis, Stat 579.001 Homework 10 Name: Answer Key
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Part I. (50 points) Chapter 10: 10.2, 10.4, 10.7, 10.12.
- For 10.4, (a) use the "pick the winner" strategy in the Taguchi analysis, (b) do a second analysis using $\bar{y}$ and $\ln(s^2)$. Comment on any differences between the two analyses in the conclusions you make.

(10^{pts}) **1. 10.2**

[ ] 10 pts

(a) (5 pts)

Solution: Yes, control variables can be used to exert influence over the variability, since there is a significant control-by-noise interaction (AB=implant dose by temperature).

(b) (5 pts)

Solution: The interaction plot (Fig. 4.9, p. 147) shows that the low setting of A will keep variation lower (spread between high and low values of B) but also keep the mean response lower, so what is desired can not be done.

(15^{pts}) **2. 10.4** - For 10.4, (a) use the "pick the winner" strategy in the Taguchi analysis, (b) do a second analysis using $\bar{y}$ and $\ln(s^2)$. Comment on any differences between the two analyses in the conclusions you make.

[ ] 15 pts

(a) (5 pts)

Solution: The Taguchi approach using "pick a winner" would select the high levels of SNR for each factor: A and C.

```
        N         Mean         Std Dev        Minimum        Maximum
a=-1    2     28.4741574      4.9322608      24.9865223     31.9617924
a= 1    2     32.0333426*     4.4294250      28.9012662     35.1654191
c=-1    2     26.9438943      2.7681420      24.9865223     28.9012662
c= 1    2     33.5636057*     2.2653061      31.9617924     35.1654191
```

```
* SAS options;
options pagesize=120 linesize=80 nodate nocenter nonumber;

* b=temperature is the noise variable, and d and e are not important;
* therefore, we consider the 4 reps over variables a and c;
data in104;
 input a c y1-y4;
 ybar = mean(of y1-y4);
 s2   = var(of y1-y4);
 lns2 = log(s2);
 snr  = -10*log10((1/y1**2 + 1/y2**2 + 1/y3**2 + 1/y4**2)/4);
cards;
-1 -1  68.7  62.1  15.1  11.3
-1  1  87.7  87.5  32.9  27.1
 1 -1 103.2 101.0  20.6  19.6
 1  1 128.3 119.0  46.1  40.3
;
run;
proc print; run;

/*
Obs   a   c    y1     y2    y3    y4   ybar      s2     lns2     snr
 1   -1  -1   68.7   62.1  15.1  11.3  39.300   917.95  6.82214  24.9865
 2   -1   1   87.7   87.5  32.9  27.1  58.800  1111.53  7.01350  31.9618
 3    1  -1  103.2  101.0  20.6  19.6  61.100  2242.31  7.71526  28.9013
```

[ ] 25 pts

```
 4    1   1  128.3  119.0  46.1  40.3  83.425  2177.42  7.68590  35.1654
*/

* pick a winner;
proc reg data=in104;  model snr=a;  run;
proc reg data=in104;  model snr=c;  run;

proc sort data=in104; by a; run;    proc means data=in104; by a; var snr; run;
proc sort data=in104; by c; run;    proc means data=in104; by c; var snr; run;

/*
       N           Mean        Std Dev        Minimum        Maximum
a=-1   2     28.4741574      4.9322608     24.9865223     31.9617924
a= 1   2     32.0333426*     4.4294250     28.9012662     35.1654191
c=-1   2     26.9438943      2.7681420     24.9865223     28.9012662
c= 1   2     33.5636057*     2.2653061     31.9617924     35.1654191
*/
```

(b) (10 pts)

Solution: The separate analysis of $\bar{y}$ and $\ln(s^2)$ shows that A and C are both significant at 0.05 for $\bar{y}$, but neither are for $\ln(s^2)$. The result is that we choose to set A and C at the high levels and then we can set the other factors at whatever levels are most convenient or cost effective.

```
* separate first-order (no interaction) models for ybar and lns2;
proc reg data=in104;
  model ybar lns2=a c;
run;
quit;

/*
Dependent Variable: ybar
                        Parameter Estimates
                     Parameter       Standard
Variable     DF       Estimate          Error     t Value    Pr > |t|
Intercept     1       60.65625        0.70625       85.88      0.0074
a             1       11.60625        0.70625       16.43      0.0387
c             1       10.45625        0.70625       14.81      0.0429

Dependent Variable: lns2
                        Parameter Estimates
                     Parameter       Standard
Variable     DF       Estimate          Error     t Value    Pr > |t|
Intercept     1        7.30920        0.05518      132.46      0.0048
a             1        0.39138        0.05518        7.09      0.0892
c             1        0.04050        0.05518        0.73      0.5969
*/
```

(10pts) **3. 10.7** [10 pts]

Solution: **For the same number of runs:** a 2^{7-1}_{VII} with $I = x_1x_2x_3x_4z_1z_2z_3$ plus 8 star points will allow unambiguous estimation of all terms of form x_i, x_ix_j, x_i^2, z_i, and x_iz_j. Degrees of freedome for error would have to be obtained from higher-order interactions.

For fewer runs: a 2^{7-2}_{IV} can be designed to estimate all main effects and control-by-noise interactions. Adding star points and center runs allows estimation of quadratic terms and gives a pure error estimate.

(15pts) **4. 10.12** [15 pts]

(a) (5 pts)

Solution: From Fig E10.2: $x_1 = -1$, x_2 doesn't matter, and $x_3 = +1$.

(b) (5 pts)

[25 pts]

Solution: From Fig E10.1: (a) $x_1 = -1$ minimizes variation due to z_1, (b) $x_2 = +1$ has slightly less variation due to z_1, (c) $x_2 = -1$ minimizes variation due to z_2, so $x_2 = +1$ should have a lower variance, and $x_3 = +1$ does not have an interaction so should be set to the level with lower response.

(c) (5 pts)

Solution: There is almost no trade-off. $x_1 = -1$ has lower response and less variability wrt z_1. $x_2 = -1$ doesn't affect the response and has slightly higher variability wrt z_1 but much less variability wrt z_2. $x_3 = +1$ has lower response, and doesn't interact with the noise variability.

0 pts