

CS 530 Geometric and Prob. Methods in CS

Homework 4

Erik Barry Erhardt

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Note: All code can be found in the Appendix.

Exercise 1 *Linear/non-linear, even, odd functions.*

(a) $f(x) = -x$.

Linear: $f(ax + y) = -(ax + y) = a(-x) + (-y) = af(x) + f(y)$

Not Even: $f(x) = -x \neq x = f(-x)$

Odd: $f(x) = -x = -(+x) = -f(-x)$

(b) $f(x) = 7x + 1$.

Not Linear: $f(ax + y) = 7(ax + y) + 1$
 $= a(7x) + 7y + 1 \neq a(7x + 1) + 7y + 1 = af(x) + f(y)$

Not Even: $f(x) = 7x + 1 \neq -7x + 1 = f(-x)$

Not Odd: $f(x) = 7x + 1 \neq -(-7x + 1) = 7x - 1 = -f(-x)$

(c) $f(x) = x^2 - x$.

Not Linear: $f(ax + y) = (ax + y)^2 - (ax + y)$
 $= a^2x^2 + 2axy + y^2 - ax - y \neq ax^2 - ax + y^2 - y = af(x) + f(y)$

Not Even: $f(x) = x^2 - x \neq x^2 + x = f(-x)$

Not Odd: $f(x) = x^2 - x \neq -x^2 - x = -f(-x)$

(d) $f(x) = e^{-x^2}$.

Not Linear: $f(ax + y) = e^{-(ax+y)^2} \neq ae^{-x^2} + e^{-y^2} = af(x) + f(y)$

Even: $f(x) = e^{-x^2} = e^{-(-x)^2} = f(-x)$

Not Odd: $f(x) = e^{-x^2} \neq -e^{-(-x)^2} = -f(-x)$

(e) $f(x) = \cos(x)$.

Not Linear: $f(ax + y) = \cos(ax + y) \neq a \cos(x) + \cos(y) = af(x) + f(y)$

Even: $f(x) = \cos(x) = \cos(-x) = f(-x)$

Not Odd: $f(x) = \cos(x) \neq -\cos(x) = -\cos(-x) = -f(-x)$

Exercise 2 *Circulant matrix function.*

See appendix.

Exercise 3 *Matlab circulant matrices and eigenvectors/eigenvalues.*

(a) **A Circulant.**

$$A = \begin{bmatrix} -1 & 4 & -3 & 2 \\ 2 & -1 & 4 & -3 \\ -3 & 2 & -1 & 4 \\ 4 & -3 & 2 & -1 \end{bmatrix}$$

(b) **Right eigenvectors/eigenvalues.**

$$W = \begin{bmatrix} -0.5 & -0.5 & -0.5 & 0.5 \\ 0.5 & -0.5i & 0.5i & 0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & +0.5i & -0.5i & 0.5 \end{bmatrix}$$

$$\lambda = \begin{bmatrix} -10 \\ 2 + 2i \\ 2 - 2i \\ 2 \end{bmatrix}$$

(c) W_{-1} and W^H are equal.

$$W^{-1} = W^H = \begin{bmatrix} -0.5 & 0.5 & -0.5 & 0.5 \\ -0.5 & +0.5i & 0.5 & -0.5i \\ -0.5 & -0.5i & 0.5 & +0.5i \\ 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}$$

$$\lambda = \begin{bmatrix} -10 \\ 2 + 2i \\ 2 - 2i \\ 2 \end{bmatrix}$$

(d) **Ay.** $y = [1234]^T$,

$$Ay = \begin{bmatrix} 6 \\ 0 \\ 14 \\ 0 \end{bmatrix}$$

(e) $W\Lambda W^H y = Ay$. $y = [1234]^T$, $\Lambda = \text{diag}(\lambda)$,

$$W\Lambda W^H y = \begin{bmatrix} 6 \\ 0 \\ 14 \\ 0 \end{bmatrix}$$

(f) $B_{100 \times 100}$ **circulant**. Like A but bigger!

(g) **Twelfth smallest right and left eigenvectors.** The real right eigenvectors 1 and 2 are the same, and similar to the left imaginary eigenvector 1, and the vertical reflection of the left imaginary eigenvector 2. Similarly, the real left eigenvectors 1 and 2 are the same, and similar to the right imaginary eigenvector 2, and the vertical reflection of the right imaginary eigenvector 1.

The plot in Figure 1 on page 4 shows the eigenvectors associated with the twelfth smallest eigenvalues plotted separately, then all together.

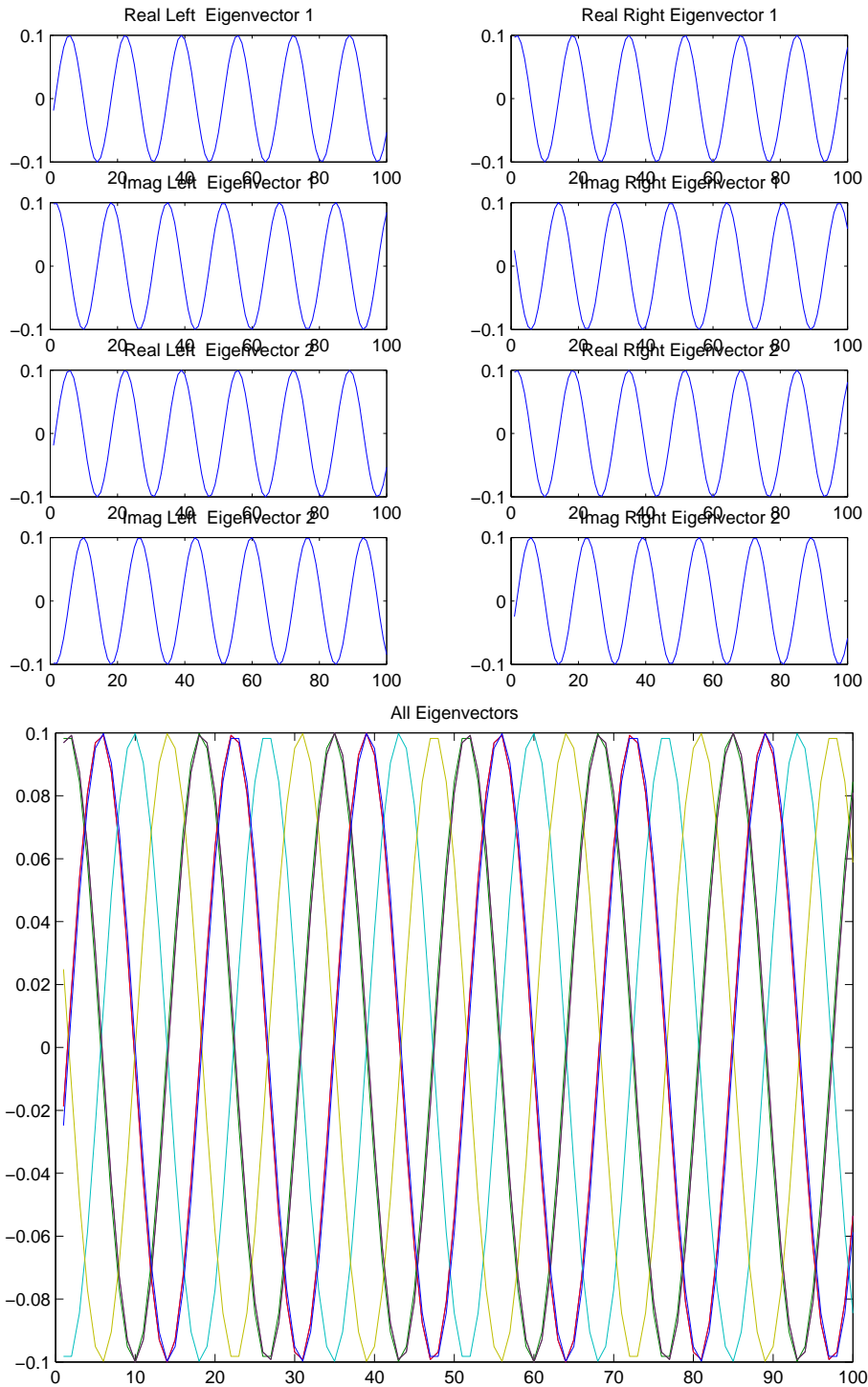


Figure 1: Problem 3g, plots of the eigenvectors associated with the twelfth smallest eigenvalues.

Exercise 4 *Proof.*

Show that $\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$, where $j = \sqrt{-1}$.

$$\begin{aligned}
 e^{jx} &= \cos(x) + j \sin(x) \\
 e^{-jx} &= \cos(-x) + j \sin(-x) \\
 e^{jx} - e^{-jx} &= \cos(x) + j \sin(x) - (\cos(-x) + j \sin(-x)) \\
 &= (\cos(x) - \cos(-x)) + j(\sin(x) - \sin(-x)) \\
 &= (\cos(x) - \cos(x)) + j(\sin(x) + \sin(x)) \quad \text{cos even, sin odd} \\
 &= 0 + 2j \sin(x) \\
 \sin(x) &= \frac{e^{jx} - e^{-jx}}{2j} \quad \text{solving for sin in last expression}
 \end{aligned}$$

Exercise 5 *Output of a linear shift-invariant system.*

Impulse response is $h(t) = \frac{\sin \pi t}{\pi t}$.

Input is $x(t) = \cos(4\pi t) + \cos(\pi t/2)$.

Output is:

$$\begin{aligned}
 y(t) &= \int_{-\infty}^{\infty} \frac{(\cos(4\pi\tau) + \cos(\pi\tau/2)) \sin(\pi(t-\tau))}{\pi(t-\tau)} d\tau \\
 &= \cos(\pi t/2) \quad \text{thank's to Maple's symbolic integration}
 \end{aligned}$$

Exercise 6 *Output of a linear shift-invariant system.*

Impulse response is $h(t) = e^{-\frac{\pi t^2}{2}}$.

Input is $x(t) = e^{j2\pi st}$.

Output is:

$$\begin{aligned}
 y(t) &= \int_{-\infty}^{\infty} e^{2j\pi s\tau} e^{-\pi/2(t-\tau)^2} d\tau \\
 &= \sqrt{2} e^{2j\pi s(js+t)}
 \end{aligned}$$

Proof of result used for Exercises 7, 8, and 11 Show $\mathcal{F}\{g(x)h(x)\} = \mathcal{F}\{g(x)\} * \mathcal{F}\{h(x)\}$.

Since I can't find this in the class notes, I decided to derive it in order to apply it later.

Show $\mathcal{F}\{g(x)h(x)\} = \mathcal{F}\{g(x)\} * \mathcal{F}\{h(x)\}$.

(Note, it is also true that $\mathcal{F}\{g(x) * h(x)\} = \mathcal{F}\{g(x)\}\mathcal{F}\{h(x)\}$.)

Let $\mathcal{F}\{g(x)\} = G(s)$ and $\mathcal{F}\{h(x)\} = H(s)$.

Convolution: $F(s) * G(s) = \int_{-\infty}^{\infty} F(y)G(s-y) dy$.

Inverse transform:

$$\begin{aligned}
 \mathcal{F}^{-1}\{\mathcal{F}\{g(x)\} * \mathcal{F}\{h(x)\}\} &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(y)G(s-y) dy e^{+j2\pi xs} ds \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(y)G(z) e^{+j2\pi x(y+z)} dy dz, \quad z = s - y, dz = ds \\
 &= \int_{-\infty}^{\infty} F(y) e^{+j2\pi x(y)} dy \int_{-\infty}^{\infty} G(z) e^{+j2\pi x(z)} dz \\
 &= f(x)g(x)
 \end{aligned}$$

Applying the Fourier transform \mathcal{F} to both sides gives what we wanted to show. ■

Exercise 7 *sine Gabor Fourier transform.*

sine Gabor: $f(t) = e^{-\pi t^2} \sin(2\pi s_0 t) = g(t)h(t)$

$F(s) = \mathcal{F}\{f(x)\} = \mathcal{F}\{g(x)h(x)\} = \mathcal{F}\{g(x)\} * \mathcal{F}\{h(x)\} = G(s) * H(s).$

$G(s) = \mathcal{F}\{g(x)\} = \mathcal{F}\{e^{-\pi t^2}\} = e^{-\pi t^2}.$

$H(s) = \mathcal{F}\{h(x)\} = \mathcal{F}\{\sin(2\pi s_0 t)\} = \frac{j}{2} [\delta(s + s_0) - \delta(s - s_0)].$

$H(s)$ is a sum of delta functions shifted by s_0 , so the convolution is

$$\begin{aligned} F(s) &= G(s) * H(s) \\ &= \frac{j}{2} [G(s + s_0) - G(s - s_0)] \\ &= \frac{j}{2} [e^{-\pi(s+s_0)^2} - e^{-\pi(s-s_0)^2}] \end{aligned}$$

Exercise 8 *Mean and Variance of two independent Normals.*

Results from statistics is the easiest way:

$$X_1 \sim N(0, \sigma_1^2)$$

$$X_2 \sim N(0, \sigma_2^2)$$

$$E(X_1 + X_2) = E(X_1) + E(X_2) = 0 + 0 = 0$$

$$\text{Var}(X_1 + X_2) = \text{Var}(X_1) + \text{Var}(X_2) + \text{Cov}(X_1, X_2) = \text{Var}(X_1) + \text{Var}(X_2) + 0 = \sigma_1^2 + \sigma_2^2$$

This can also be done by taking $Y = X_1 + X_2$ and performing a convolution. However, there is a great deal of algebra required to complete the square in the exponent.

Alternatively we can take the Fourier transform of the convolution, take the product in frequency domain, then take the inverse Fourier transform.

Exercise 9 *Fourier transform of a location-scale Pulse.*

$$f(t) = \begin{cases} 1, & \text{if } |at - b| \leq \frac{1}{2} \\ 0, & \text{else} \end{cases}$$

$$\begin{aligned} F(s) &= \int_{-\infty}^{\infty} f(t)e^{-j2\pi st} dt \\ &= \int_{\frac{-\frac{1}{2}+b}{a}}^{\frac{\frac{1}{2}+b}{a}} e^{-j2\pi st} dt \\ &= \frac{1}{-j2\pi s} \left[e^{-j2\pi s \frac{\frac{1}{2}+b}{a}} - e^{-j2\pi s \frac{-\frac{1}{2}+b}{a}} \right] \end{aligned}$$

A location-scale transform of the sinc.

Exercise 10 *Inverse Fourier transform.*

$$\begin{aligned} g(t) &= \int_{-\infty}^{\infty} e^{j2\pi s_0 \tau} h(t - \tau) d\tau \\ &= H(s_0)e^{j2\pi s_0 t} \\ &= \frac{1}{s_0} e^{j2\pi s_0 t} \end{aligned}$$

Exercise 11 *similar to cosine Gabor, but first derivative of Gaussian.*

$$\text{Function: } f(t) = -2\pi t e^{-\pi t^2} \cos(2\pi s_0 t) = g(t)h(t)$$

$$F(s) = \mathcal{F}\{f(x)\} = \mathcal{F}\{g(x)h(x)\} = \mathcal{F}\{g(x)\} * \mathcal{F}\{h(x)\} = G(s) * H(s).$$

$G(s) = \mathcal{F}\{g(x)\} = \mathcal{F}\{-2\pi t e^{-\pi t^2}\} = \mathcal{F}\left\{\frac{d}{dt} e^{-\pi t^2}\right\} = (js)^1 e^{-\pi t^2}$, by the generalized derivative property of the Fourier transform¹.

$$H(s) = \mathcal{F}\{h(x)\} = \mathcal{F}\{\cos(2\pi s_0 t)\} = \frac{1}{2} [\delta(s + s_0) + \delta(s - s_0)].$$

$H(s)$ is a sum of delta functions shifted by s_0 , so the convolution is

$$\begin{aligned} F(s) &= G(s) * H(s) \\ &= \frac{1}{2} [G(s + s_0) + G(s - s_0)] \\ &= \frac{1}{2} \left[j(s + s_0) e^{-\pi(s+s_0)^2} + j(s - s_0) e^{-\pi(s-s_0)^2} \right] \\ &= \frac{j(s + s_0)}{2} \left[e^{-\pi(s+s_0)^2} + e^{-\pi(s-s_0)^2} \right] \end{aligned}$$

¹http://en.wikipedia.org/wiki/Fourier_transform

Exercise 12 Show if $\mathcal{F}\{f\}(s) = F(s)$ then $\mathcal{F}\{F\}(s) = f(-s)$.

Def: $f(t) \xrightarrow{\mathcal{F}} F(s)$.

Basically, show $\mathcal{F}\{\mathcal{F}\{f\}\}(s) = f(-s)$.

$$\begin{aligned}
 \mathcal{F}\{f\}(s) &= \langle e^{j2\pi st}, f \rangle \\
 &= \int_{-\infty}^{\infty} f(s)e^{j2\pi st} ds \\
 &= F(s) \\
 \mathcal{F}\{F\}(s) &= \int_{-\infty}^{\infty} F(s)e^{-j2\pi st} ds \\
 &= \int_{-\infty}^{\infty} F(-u)e^{-j2\pi(-u)t} \left| \frac{du}{ds} \right| ds \quad u = -s, s = -u, ds = -du, \frac{du}{ds} = -1 \\
 &= \int_{-\infty}^{\infty} F(-u)e^{j2\pi ut} du \\
 &= f(-u)
 \end{aligned}$$

Exercise 13 Show $\mathcal{F}^{-1}\{\mathcal{F}\{f\}\} = f$.

$$\begin{aligned}
 \mathcal{F}\{f\} &= \langle e^{j2\pi st}, f \rangle \\
 &= \int_{-\infty}^{\infty} f(s)e^{j2\pi st} ds \\
 &= F(s) \\
 \mathcal{F}^{-1}\{F\} &= \int_{-\infty}^{\infty} F(s)e^{+j2\pi st} ds \\
 &= f(t)
 \end{aligned}$$

Or, by definition, $\mathcal{F}^{-1}\{\mathcal{F}\{f\}\} = (\mathcal{F}^{-1} \circ \mathcal{F})\{f\} = 1\{f\} = f$.

Exercise 14 *truncated Fourier series.*

The plot in Figure 2 on page 11 shows truncated Fourier series of

$$f_N(t) = \pi + \sum_{\substack{\omega = -N \\ \omega \neq 0}}^N \frac{j}{\omega} e^{j\omega t}$$

for $N = 1, 3, 6, 12, 24, 48$. Note that the imaginary part is always the constant 0.

This shows why the imaginary part is a constant 0, because the positive and negative imaginary parts cancel:

$$\begin{aligned} \frac{j}{\omega} e^{j\omega t} + \frac{j}{-\omega} e^{-j\omega t} &= \frac{j}{\omega} [\cos(\omega t) + j \sin(\omega t)] + \frac{j}{-\omega} [\cos(-\omega t) + j \sin(-\omega t)] \\ &= \frac{j}{\omega} \cos(\omega t) + \frac{j}{-\omega} \cos(-\omega t) + \frac{j^2}{\omega} \sin(\omega t) + \frac{j^2}{-\omega} \sin(-\omega t) \\ &= 0 + \frac{-1}{\omega} \sin(\omega t) + \frac{-1}{-\omega} \sin(-\omega t), \quad \text{cancel since cos is even} \\ &= -\frac{1}{\omega} \sin(\omega t) + \frac{1}{\omega} \sin(-\omega t) \\ &= -\frac{1}{\omega} \sin(\omega t) - \frac{1}{\omega} \sin(\omega t), \quad \text{sign change since sin is odd} \\ &= -\frac{2}{\omega} \sin(\omega t) \end{aligned}$$

Therefore, only a real sine.

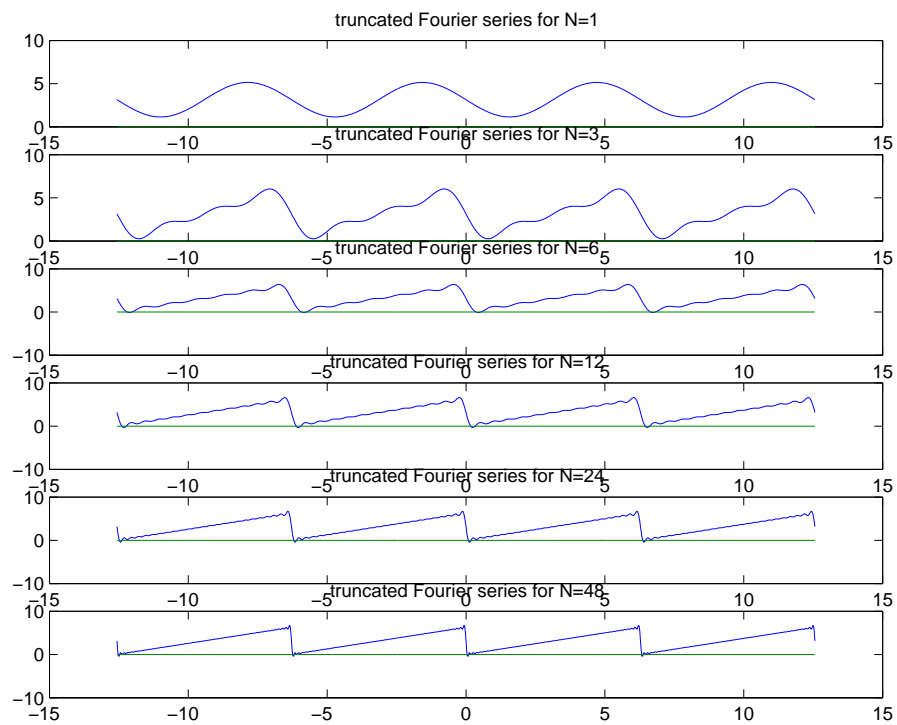


Figure 2: Problem 14, Plot of truncated Fourier series.

Appendix

Matlab code used for the exercises

*** Exercise 2 =====

Listing 1: Matlab routine `circ.m` used to convert an $N \times 1$ vector x to a circulant matrix A .

```

function [A] = circ(x)
% function [A] = circ(x)
%
% Construct a circulant matrix out of an nxl vector
%
5  % Erik Barry Erhardt 11/6/2006
%
% error checking
if nargin ~= 1; error('Only one input argument!'); return; end;
10 isVector = ((ndims(x) == 2) && ((size(x,1) == 1) || (size(x,2) == 1)));
if ~isVector; error('x is not a vector!'); return; end;

if size(x,1) < size(x,2); x=x'; end; % make a column vector

15 len_x = max(size(x)); % length of the vector
A = zeros(len_x); % init matrix A

for i = 0:(len_x-1);
20 ind = mod((1:len_x)-i-1,len_x)+1; % index order
A(:,i+1)=x(ind); % write the rows of x in the order of ind
end; %for

% EOF %

```

*** Exercise 3 =====

```

tol=1e8;

% (a)
xind = [1:4]';
x = xind.*(-1).^xind;
A = circ(x) % create circulant matrix A

% (b)
[W,lambda] = eig(A)
dlambda = diag(lambda)
W = round(W*tol)/tol

% (c)
% these are equal
inv(W)
W'

% (d)
y=[1:4]';
round(A*y *tol)/tol

% (e)
round(W*lambda*W'*y *tol)/tol

% (f)
xind = [1:100]';
x = xind.*(-1).^xind;
B = circ(x); % create circulant matrix B

% (g)
% right
twelfth_smallest_ind=100-12+1;
[RW,Rlambda] = eig(B); % RW is right eigenvectors
RW=round(RW*tol)/tol;

dRlambda = diag(Rlambda);
dRlambda_abs = abs(dRlambda); % magnitude of eigenvalues
[dRlambda_abs_sort, dRlambda_abs_ind] = sortrows(dRlambda_abs);
dRlambda_abs_sort = flipud(dRlambda_abs_sort);
dRlambda_abs_ind = flipud(dRlambda_abs_ind);

[dRlambda_abs_sort(twelfth_smallest_ind), dRlambda_abs_ind(twelfth_smallest_ind)] % 12th smallest eigenvalue
% 50.9016074095522 95

```

```

R12_ind = dRlamba_abs_ind(find(dRlamba_abs_sort==dRlamba_abs_sort(twelfth_smallest_ind))); % indicies of 12th smallest eigenvalue
Revec_12 = RW(:,R12_ind);

% left
[LW,Llamba] = eig(B. '); LW = conj(LW); % LW is left eigenvectors
LW=round(LW*tol)/tol;

dLlamba = diag(Llamba);
dLlamba_abs = abs(dLlamba); % magnitude of eigenvalues
[dLlamba_abs_sort, dLlamba_abs_ind] = sortrows(dLlamba_abs);
dLlamba_abs_sort = flipud(dLlamba_abs_sort);
dLlamba_abs_ind = flipud(dLlamba_abs_ind);

[dLlamba_abs_sort(twelfth_smallest_ind), dLlamba_abs_ind(twelfth_smallest_ind)] % 12th smallest eigenvalue
% 50.9016074095521 86
L12_ind = dLlamba_abs_ind(find(dLlamba_abs_sort==dLlamba_abs_sort(twelfth_smallest_ind))); % indicies of 12th smallest eigenvalue
Levec_12 = LW(:,L12_ind);

% plots
subplot(4,2,1);plot(xind,real(Levec_12(:,1)));title('Real Left Eigenvector 1');
subplot(4,2,3);plot(xind,imag(Levec_12(:,1)));title('Imag Left Eigenvector 1');
subplot(4,2,5);plot(xind,real(Levec_12(:,2)));title('Real Left Eigenvector 2');
subplot(4,2,7);plot(xind,imag(Levec_12(:,2)));title('Imag Left Eigenvector 2');
subplot(4,2,2);plot(xind,real(Revec_12(:,1)));title('Real Right Eigenvector 1');
subplot(4,2,4);plot(xind,imag(Revec_12(:,1)));title('Imag Right Eigenvector 1');
subplot(4,2,6);plot(xind,real(Revec_12(:,2)));title('Real Right Eigenvector 2');
subplot(4,2,8);plot(xind,imag(Revec_12(:,2)));title('Imag Right Eigenvector 2');
plot_name = strcat('gpcshw04-3indy.eps');
print(gcf, '-depsc2', plot_name);

clf;plot(xind,[real(Levec_12(:,1)),imag(Levec_12(:,1)),real(Levec_12(:,2)),imag(Levec_12(:,2)),real(Revec_12(:,1)),imag(Revec_12(:,1)),real(Revec_12(:,2)),imag(Revec_12(:,2))]);
title('All Eigenvectors');
plot_name = strcat('gpcshw04-3all.eps');
print(gcf, '-depsc2', plot_name);

%%% Exercise 4 =====
%%% Exercise 5 =====

% Maple code:
f:=(cos(4*pi*tau)+cos(pi*tau/2))*sin(pi*(t-tau))/(pi*(t-tau));
> latex(f);
      (cos(4 pi tau) + cos(1/2 pi tau)) sin(pi (t - tau))
f := -----
      pi (t - tau)

{\frac { \left( \cos \left( 4 \, \pi \, \tau \right) + \cos \left( 1/2 \, \pi \, \tau \right) \right) \sin \left( \pi \, \left( t - \tau \right) \right) }{ \pi \, \left( t - \tau \right) }}
> F:=simplify(int(f,tau=-infinity..infinity));
> latex(F);
      Pi cos(1/2 pi t)
F := -----
      | pi |

{\frac {\pi \, \cos \left( 1/2 \, \pi \, t \right) }{ \left| \pi \right| }}

%%% Exercise 6 =====

% Maple code:
f:=exp(sqrt(-1)*pi*s*tau)*exp(-pi/2*(t-tau)^2);
> latex(f);
      2
f := exp(2 I pi s tau) exp(- 1/2 pi (t - tau) )

{e^{2,i\pi s\tau}}{e^{-1/2,\pi}, \left( t-\tau \right) ^{2}}
> F:=int(f,tau=-infinity..infinity);
> latex(F);
      {
      { exp(2 I pi s (I s + t)) 2 1/2 Pi
      { ----- csgn(pi) = 1
      { 1/2
      { pi
      {
      { infinity otherwise

\cases{
{\frac {{e^{2,i\pi s} \left( is+t \right) }}{\sqrt {2}\sqrt {\pi }}}{\sqrt {\pi }}}
&{\it csgn} \left( \pi \right) =1\cr \infty &{\it otherwise}\cr

```

```

=====  

### Exercise 7  

% Maple code:  

> f:=1/sqrt(2*pi)*sin(2*pi*s*t)*exp(-pi*t^2-2*pi*sqrt(-1)*omega*t);  

> latex(f);  


$$f := \frac{1}{2} \frac{\sin^2(\pi s t) \exp(-\pi t^2 - 2 I \pi \omega t)}{\pi}$$

1/2\,{\frac {\sqrt {2}\sin \left( 2\,\pi \right) \,st \right) {e^{-\pi},\{t\}^2}  

-2\,i\pi,\,\omega,t}}{\sqrt {\pi}}  

> F:=simplify(int(f,t=-infinity..infinity));  

> latex(F);  


$$F := \frac{(\cos(4 \pi \tau) + \cos(1/2 \pi \tau)) \pi}{|\pi|}$$

{\frac { \left( \cos \left( 4\,\pi \right) \,\tau \right) +\cos \left( 1/2\,\pi \right) \,  

\,\tau \right) \right) \pi }{|\pi|}
  

=====  

### Exercise 8  

=====  

### Exercise 9  

=====  

% Maple code:  

> f:=exp(-2*pi*sqrt(-1)*omega*t)/sqrt(2*pi);  

> latex(f);  


$$f := \frac{1}{2} \frac{\exp(-2 I \pi \omega t)}{\pi}$$

1/2\,{\frac {{e^{-2\,i\pi},\,\omega,t}}{\sqrt {2}}\sqrt {\pi}}  

> F:=simplify(int(f,t=(-1/2+b)/a..(1/2+b)/a));  

> latex(F);  


$$F := -1/4 I \frac{\exp(-I \pi \omega (1 + 2 b) / a) - \exp(-I \pi \omega (-1 + 2 b) / a)}{(\pi \omega)^{3/2}}$$

-1/4\,i\sqrt {2} \left( -\frac {e^{-i\omega},\,\pi}, \left( 1+2\,b \right) \right) \{a\} + e^{-i\omega},\,\pi, \left( -1+2\,b \right) \}  

a}} \right) \pi^{-3/2}\{\omega\}^{-1}
  

=====  

### Exercise 10  

=====  

### Exercise 11  

=====  

f:=-2*pi*t*exp(-pi*t^2)*cos(2*pi*s*t)*exp(-2*pi*sqrt(-1)*s*t);  

> latex(f);  


$$f := -2 \pi t \exp(-\pi t^2) \cos(2 \pi s t) \exp(-2 I \pi s t)$$

-2\,\pi,\,t\{e^{-\pi},\{t\}^2}\}\cos \left( 2\,\pi \right) \,st \right) {e^{-2\,i  

\pi},st}  

> F:=simplify(int(f,t=-infinity..infinity));  

> latex(F);  


$$F := \int_{-\infty}^{\infty} -2 \pi t \exp(-\pi t^2) \cos(2 \pi s t) dt$$

\int _{-\infty }^{\infty } \{-2\,\pi,\,t\{e^{-\pi},t \left( t+2\,i  

s \right) \}\cos \left( 2\,\pi \right) \,st \right) \{dt\}
  

=====  

### Exercise 12  

=====

```

```

*** Exercise 13 =====
*** Exercise 14 =====

```

Listing 2: Matlab routine `fourier_series_truncated.m` used to compute the truncated Fourier series of a specific function hard-coded.

```

function [f_t] = fourier_series_truncated(N,t)
% function [f_t] = fourier_series_truncated(N)
%
% Computes the truncated Fourier series of a specific function hard-coded
5 %
% N is number of frequencies
% t is a linspace points from -4*pi to 4*pi (or any range)
% Erik Barry Erhardt 11/6/2006

10 % error checking
if nargin ~= 2; error('Two input arguments!'); return; end;

f_t = zeros(length(t),1); % init function being approximated
f_t = f_t + pi;           % starts as pi

15 j=sqrt(-1);
for omega=1:N;
    term_pos = (j/ omega )*exp(j* omega *t);
    term_neg = (j/(-omega))*exp(j*(-omega)*t);
20 f_t = f_t + term_pos + term_neg;
end; %for

25 % EOF %

t=linspace(-4*pi,4*pi,1001)';
N=[1,3,6,12,24,48];
plot_ind=0;
for i_N = N;
    plot_ind=plot_ind+1;
    [f_t] = fourier_series_truncated(i_N,t);
    subplot(length(N),1,plot_ind);plot(t,[real(f_t),imag(f_t)]);
    title(strcat('truncated Fourier series for N=',num2str(i_N)));
end;
plot_name = strcat('gpcshw04-14.eps');
print(gcf, '-depsc2', plot_name);

```