Name: Key

STAT 345 - Summer, 2006 - Exam 1

Based on sections: 2.1 – 2.6, 2.8, 3.1 – 3.4

You may use a calculator and an $8\frac{1}{2}$" by 11" sheet of paper.

Show all work to receive full credit.

1. If $P(A) = 0.7, P(B) = 0.8$ and $P(A \cap B) = 0.6$ determine the following probabilities: (4 points each)

(a) $P(A')$
$= 1 - P(A)$
$= 1 - 0.7 = 0.3$

(b) $P(A \cup B)$
$= P(A) + P(B) - P(A \cap B)$
$= 0.7 + 0.8 - 0.6$
$= 0.9$

(c) $P(A' \cap B) = P(B) - P(A \cap B)$
$= 0.8 - 0.6$
$= 0.2$

(d) $P[(A \cup B)']$
$= 1 - P(A \cup B)$
$= 1 - 0.9$
$= 0.1$

(e) $P(A' \cup B)$
$= P(A') + P(B) - P(A' \cap B)$
$= 0.3 + 0.8 - 0.2$
$= 0.9$

2. A batch of 100 light bulbs contains 12 that are defective. Two light bulbs are selected at random, without replacement, from the batch. Find the following. (3 points each)

   (a) How many outcomes are in the resulting sample space?

   Two are selected without replacement $\Rightarrow$ there are 100 choices for the first selection and because it is without replacement, there are 99 left for the second selection. This gives $(100)(99) = 9900$ outcomes.

   (b) Find the probability that the first light bulb is defective.

   There are 100 light bulbs and 12 of these are defective.

   $$\Rightarrow \frac{12}{100}$$

   (c) Find the probability that the second light bulb is not defective given the first light bulb is defective.

   As we are sampling without replacement, there are 99 light bulbs left after selecting the first. Of these, 88 are not defective as the first one was defective.

   $$\Rightarrow \frac{88}{99}$$

3. Suppose 4% of cotton fabric rolls and 1% of nylon fabric rolls contain flaws. Of the rolls used by a manufacturer, 80% are cotton and 20% are nylon. What is the probability that a randomly selected roll used by the manufacturer contains flaws? (5 points)

   $P(\text{flawed}|\text{cotton}) = 0.04$
   $P(\text{flawed}|\text{nylon}) = 0.01$
   $P(\text{cotton}) = 0.80$
   $P(\text{nylon}) = 0.20$
   $\Rightarrow P(\text{flawed}) = P(\text{flawed}|\text{cotton})P(\text{cotton}) + P(\text{flawed}|\text{nylon})P(\text{nylon})$
   $= (0.04)(0.80) + (0.01)(0.20)$
   $= 0.034$

4. Let A and B be two events such that $P(A) = 0.3$, $P(B) = 0.6$, and $P(A \cap B) = 0.1$. (5 points each)

   (a) What is $P(A|B)$?

   $$= \frac{P(A \cap B)}{P(B)}$$

   $$= \frac{0.1}{0.6}$$

   $$= 0.1667$$

   (b) Are A and B independent events? Explain why or why not?
   No, because $P(A|B) \neq P(A)$, as was shown in part (a).

5. An unmanned intruder detection system was tested outdoors under various weather conditions in Tokyo, Japan, as described in *IEEE Computer Applications in Power*. The numbers of intruders detected and missed under each condition are provided in the table below:

	Weather condition				
	Clear	Cloudy	Rainy	Snowy	Windy
Intruders detected	21	228	226	7	185
Intruders missed	0	6	6	3	10

   (a) Under snowy conditions, what is the probability that the unmanned system detects an intruder? (3 points)

   Under snowy conditions there were 10 intruders and 7 of these were detected.
   $\Rightarrow P(\text{detected}|\text{snowy}) = \frac{7}{10}$

   (b) Given that the unmanned system missed detecting an intruder, what is the probability that the weather condition was windy?

   There were 25 intruders missed by the system. Of these, 10 were under windy conditions.
   $\Rightarrow P(\text{windy}|\text{missed}) = \frac{10}{25}$

6. In a semiconductor manufacturing process, three wafers from a lot are tested. Each wafer is classified as pass or fail. Assume that the probability that a wafer passes the test is 0.82 and that wafers are independent. Determine the probability mass function of the number of wafers out of three from a lot that pass the test. (10 points)

   Let f denote a wafer that fails the test and p a wafer that passes. Then x has range $\{0, 1, 2, 3\}$ where $x = 0$ corresponds to $\{fff\}$, $x = 1$ corresponds to $\{\{ffp\}, \{fpf\}, \{pff\}\}$, $x = 2$ corresponds to $\{\{fpp\}, \{pfp\}, \{ppf\}\}$, and $x = 3$ to $\{ppp\}$. Probabilities of individual events can be computed using independence, $P(p) = 0.82$ and $P(f) = 0.18$. For instance, $P(fff) = (0.18)(0.18)(0.18)$ and $P(fpf) = (0.18)(0.82)(0.18)$. Doing this for each event and summing the probabilities for the events with $x = 1$ and $x = 2$ gives the distribution:

x	$f(x)$
0	$(0.18)^3$
1	$3(0.82)(0.18)^2$
2	$3(0.82)^2(0.18)$
3	$(0.82)^3$

7. Suppose the probability mass function of a random variable X is given by $f(x) = \dfrac{2x+1}{k}$, $x = 0, 1, 2, 3, 4$. Find the following: (4 points each)

(a) Determine the constant k so that $f(x)$ is a pmf.

$$\sum_{x=0}^{4} f(x) = 1$$

$$\Rightarrow \frac{2(0)+1}{k} + \frac{2(1)+1}{k} + \frac{2(2)+1}{k} + \frac{2(3)+1}{k} + \frac{2(4)+1}{k} = 1$$

$$\Rightarrow \frac{1}{k} + \frac{3}{k} + \frac{5}{k} + \frac{7}{k} + \frac{9}{k} = 1$$

$$\Rightarrow \frac{25}{k} = 1$$

$$\Rightarrow k = 25$$

(b) $P(X = 4)$

$$= \frac{2(4)+1}{25} = \frac{9}{25} = 0.36$$

(c) $P(X \leq 1)$

$P(X = 0) + P(X = 1) = \dfrac{2(0)+1}{25} + \dfrac{2(1)+1}{25} = \dfrac{1}{25} + \dfrac{3}{25} = \dfrac{4}{25} = 0.16$

(d) $P(2 \leq X < 4)$

$$= P(X = 2) + P(X = 3) = \frac{5}{25} + \frac{7}{25} = \frac{12}{25} = 0.48$$

(e) $P(X > 0)$

$$= 1 - P(X = 0) = 1 - \frac{1}{25} = \frac{24}{25} = 0.96$$

8. Consider the discrete random variable X with finite range $\{-2, -1, 0, 1\}$ and probability mass function:

x_i	$f(x_i)$
-2	0.2
-1	0.3
0	0.4
1	0.1

(a) Find $E(X)$. (5 points)

$$E(X) = \sum_x x f(x)$$

$$= (-2)(0.2) + (-1)(0.3) + (0)(0.4) + (1)(0.1)$$

$$= -0.4 - 0.3 + 0 + 0.1$$

$$= -0.6$$

(b) Find $Var(X)$. (5 points)

$$Var(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum_x x^2 f(x) = (-2)^2(0.2) + (-1)^2(0.3) + (0)^2(0.4) + (1)^2(0.1) = 0.8 + 0.3 + 0 + 0.1 = 1.2$$

$$\Rightarrow Var(X) = 1.2 - (-0.6)^2 = 1.2 - 0.36 = 0.84$$

(c) Find the cumulative distribution function $F(x) = P(X \leq x)$. (10 points)

$$F(X) = \begin{cases} 0.0, & x < -2 \\ 0.2, & -2 \leq x < -1 \\ 0.5, & -1 \leq x < 0 \\ 0.9, & 0 \leq x < 1 \\ 1.0, & 1 \leq x \end{cases}$$