STAT 345 - Examples

Based on all chapters and sections

Introduction

0.1 Populations and Samples

Ex 1: Research engineers with the University of Kentucky Transportation Research Program have collected data on accidents occurring at intersections in Lexington, Kentucky, over a period of 5 years. One of the goals of the study was to compare the average number of left-turn accidents at locations with and without left-turn-only lanes, in order to develop guidelines for the installation of left-turn lanes.

Ex 2: Researchers have developed a new pre-cooling method for preparing vegetables for the market. The system employs an air and water mixture designed to yield effective cooling with a much lower water flow than conventional hydrocooling. In an effort to compare the effectiveness of the two cooling systems, 20 batches of green tomatoes were divided into two groups; one group was pre-cooled with the new method, and the other with the conventional method. The total water flow (in gallons) required to effectively cool each batch was recorded.

0.2 Descriptive Statistics

Ex 1: The FDA sets limits for DDT content in fish at 5 parts per million (ppm). Fish with DDT content exceeding this limit are considered potentially hazardous if consumed. In the summer of 1980 the U.S. Army Corps of Engineers collected fish specimens at different locations along the Tennessee River downstream from an inactive manufacturing plant to determine if discharged toxic material contaminated the fish. 144 fish were captured and the DDT concentration (in ppm) was measured in each fish. A histogram of the data is given on the next page.

The mean is 24.35 ppm. The standard deviation is 98.38.

Chapter 2

2.1 Sample Spaces and Events

Ex 1: An experiment involves finding the proportion of insects killed on a plant after an insecticide is applied. What is the sample space?

Ex 2: Emissions from 3 suppliers are classified for conformace to air quality specifications. The results from 100 samples are in the table below.

Supplier	Conforms Yes	Conforms No
1	22	8
2	25	5
3	30	10

A=event that sample is from Supplier 1
B=event that sample conforms
How many samples are in each of these sets?
a) $A \cap B =$
b) $A' \cap B =$
c) $B' =$

Ex 3: A random experiment involves measuring the lifetime of an electrical component (in hours). Let A be the event that the component lasts no longer than 24 hours. Let B be the event that it lasts longer than 12 hours.
Find $S, A, B, A', B', A \cap B$.

2.3 Additional Rules

Ex 1: Records at an industrial plant show that
12% of injured workers are admitted to hospital,
16% of injured workers are back on the job the next day, and
2% of injured workers are both admitted to hospital and back on the job the next day.
What is the probability an injured worker is either admitted to hospital, back on the job the next day, or both?
A=event admitted to hospital
B=event back to work the next day

2.4 Conditional Probability

Ex 1: Samples of galvanized steel are analyzed for coating weight and surface roughness.

Surface Roughness	Coating Weight High	Coating Weight Low	
High	12	16	28
Low	88	34	122
	100	50	150

A=event coating wt high
B=event surface roughness high
a) Find $\Pr(B|A)$, i.e. Probability surface high given weight high.
b) Find $\Pr(A'|B)$, i.e. Probability coating not high (coating low) given surface high.

Ex 2: Suppose a batch contains 50 parts, 10 of which are defective. 2 parts are selected at random without replacement. What is the probability that the second part is not defective given that the first part is defective? Solve by reasoning and by formula.

2.5 Multiplication and Total Probability Rule

Ex 1: A two-component electrical system is connected in parallel, so that it fails only if both components fail. The probability that the first fails is 0.10. If the first fails, the probability the second fails is 0.05. What is the probability that the system fails?

Ex 2: The probability is 1% (0.01) that an electrical connector that is kept dry fails during the warranty period of a portable computer. If the connector is ever wet, the probability of failure during the warranty period is 5% (0.05). If 90% of connectors are kept dry, what proportion are expected to fail during the warranty period? Find Pr(fail).

2.6 Independence

Ex 1: A production line is being monitored by a quality control engineer who discovers 20% of items being produced are defective.
a) If two items arrive in succession off the line, what is the probability both are defective? Assume conditions of the two items are independent.
b) If ten items arrive in succession off the line, what is the probability all are defective?

2.7 Bayes' Theorem

Ex 1: A TB test is administered to a randomly chosen person. 0.1% of the population have TB. The probability the test is positive given he has TB is 0.98. The probability the test is positive given he does not have TB is 0.05.
a) What is the probability of testing positive?
b) If he does test positive, what is the probability he has TB?
c) Is this an effective TB test?

2.8 Random Variables

Ex 1: Conduct an opinion poll by asking 100 randomly selected people their opinion on an issue and record 1 or 0 for agree or disagree. a) What is the sample space?
b) How many elements in the sample space?
c) Suppose we care only about the number of people who agree. What is the sample space and how many elements does it have?

Chapter 3

3.2 Probability Distributions and PMFs

Ex 1: Two parts selected at random from a batch are tested to see if they meet manufacturer's specifications. Assume that the probability a part meets specs is 0.93 and that parts are independent. Let X be the number of parts meeting specs. Find the distribution of X.

3.6 Bernoulli and Binomial Distributions

Ex 1: A solar heating panel was designed to have a life of at least 5 years with probability 0.95. A random sample of 20 was selected and lifetime of each was recorded.
a) What is the probability that exactly 18 have a lifetime of at least 5 years?
b) What is the probability at most 10 have a 5 year lifetime?
c) If only 10 had a 5 year lifetime, what would you say about p?

3.7 Geometric and Negative Binomial Distributions

Ex 1: An electronic scale in an automated filling operation stops the manufacturing line after 3 underweight packages are selected. Suppose the probability of an underweight package is 0.002, and each fill is independent. What is the mean number of fills before the line is stopped?

3.9 Poisson Distribution

Ex 1: Suppose that the number of cracks per concrete specimen has a Poisson distribution with 2.5 cracks per specimen.
a) Find the probability that a randomly selected specimen has exactly 5 cracks.
b) Find the probability it has 2 or more.

Chapter 4

4.2 Continuous RVs and Probability Distributions

Ex 1: Suppose X is the lifetime of an electrical component in years and the PDF of X is $f(x) = \frac{1}{4}e^{-x/4}, x \geq 0$.
a) Verify $f(x)$ is a PDF.
b) Find the probability that a randomly selected component lasts longer than 3 years.
c) Find the probability it lasts between 1 and 2 years.

4.3 CDFs

Ex 1: The PDF for a RV X is given by $f(x) = cx^2, 0 \leq x \leq 2$.
a) Find c so that $f(x)$ is a pdf.
b) Find $\Pr(X \leq 3)$.
c) Find the cdf of $f(x)$.

Ex 2: Suppose RV X has pdf $f(x) = 6(x - x^2), 0 \leq x \leq 1$.
a) Show $\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x = 1$.
b) Find $\Pr(X > \frac{1}{2})$.
c) Find the cdf.

4.5 Continuous Uniform Distribution

Ex 1: The thickness of steel sheets from rolling machines have a uniform distribution with values between 150mm-200mm any sheets with thickness less than 160mm are scrapped.
a) Calculate the fraction scrapped.
b) Find the mean and std.dev. of thickness.

4.9 Exponential Distribution

Ex 1: Suppose the number of arrivals at a service counter follows a Poisson distribution with mean number of arrivals $\lambda = 3$ per hour. Then the length of time between 2 successive arrivals, X, has $X \sim \text{Exp}(3), f(x) = 3e^{-3x}, x \geq 0$. Find the mean, variance, and standard deviation of X.

Ex 2: The time until failure, in hours, of a fan in a computer can be modelled as Exp(0.0003).
a) What proportion of fans will last at least 10,000 hours?
b) What proportion of fans will last at most 7,000 hours?
c) What is a fan's expected lifetime?
d) Would you be happy with one of these fans in your computer?

Chapter 8

8.2 CI of Mean of Normal Distribution, Known Variance

Ex 1: Soft drink cans are filled by an automated filling machine. Assume that the fill volumes of cans are independent, normal RVs, with mean $\mu = 12.1$ fl.oz. and std.dev. $\sigma = 0.5$ fl.oz.
a) What is the distribution of the average fill volume of $n = 100$ cans?
b) What is the probability the average fill volume is less than 12 fl.oz.?
c) What is the probability the average fill volume is between 11.9 and 12.3 fl.oz.?

Ex 2: Biologists studying the healing of skin wounds measured the rate at which new cells closed on a razor cut made in the skin of an anesthetized neut. The data for $n = 18$ neuts in micrometers/hour:
29, 27, 34, 40, 22, 28, 16, 35, 26, 35, 12, 30, 23, 18, 11, 22, 23, 33
Assuming the neuts are a random sample from their species, and that the population standard deviation is 4 micrometers/hour, find a 90% CI for the mean healing rate of neuts.