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CH 9 HYPOTHESIS TESTING

9.1

A HYPOTHESIS TEST IS USED TO MAKE A DECISION ABOUT A POPULATION PARAMETER.

[HANDOUT 10] #1. "WANT" $\mu = 1.00$ IN, $\mu \neq 1.00$ IS "UNDESIRED"

DENOTE

$H_0: \mu = 1.00$ NULL HYPOTHESIS

$H_a: \mu \neq 1.00$ ALTERNATIVE HYPOTHESIS

WE TEST H_0 VS. H_a.

THE RESULT WILL SUGGEST SOME COURSE OF ACTION.

IN OUR EXAMPLE, IF WE REJECT H_0 IN FAVOR OF H_a, WE WILL HAVE TO FIX THE MACHINE.

TO PERFORM TEST: WE TAKE A RANDOM SAMPLE AND SEE WHICH OF THE TWO HYPOTHESES OUR DATA IS MOST CONSISTENT WITH.

THE STRATEGY IS "INNOCENT UNTIL PROVEN GUILTY" FOR H_0.

IF "GUILTY", WE "REJECT H_0 IN FAVOR OF H_a".

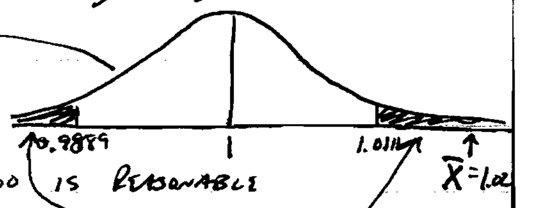
IF "NOT GUILTY," WE "FAIL TO REJECT H_0".

#1. $n = 50$ BOLTS, $\bar{X} = 1.02''$, $S = 0.04''$

IS $\bar{X} = 1.02$ CLOSE ENOUGH TO $\mu = 1$?

IF THE MEAN LENGTH OF POPULATION IS TRULY $\mu = 1$,

THEN $\bar{X} \sim N(1.00, \frac{\sigma^2}{n}) = N(1, \frac{.04^2}{50})$.



SET UP A $100(1-\alpha)\%$ CI ABOUT $\mu = 1.00$.

IF OBSERVED $\bar{X}$ FALLS IN THE INTERVAL, $\mu = 1.00$ IS REASONABLE SO WE "FAIL TO REJECT H_0".

IF OBSERVED $\bar{X}$ FALLS OUTSIDE THE INTERVAL IN THE REJECTION REGION, WE "REJECT H_0 IN FAVOR OF H_a".

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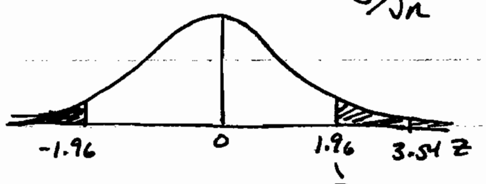
STANDARDIZE TO $N(0,1)$

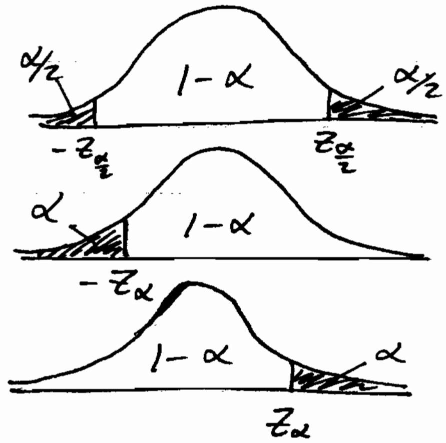
$$z = \frac{1.02 - 1}{.04/\sqrt{50}} = 3.54$$

Z-SCORE IS $z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$, THE STANDARDIZED OBSERVED VALUE.



SINCE Z IS IN THE REJECTION REGION, WE REJECT H_0 IN FAVOR OF H_a, CONCLUDING THAT $\mu \neq 1$.

POSSIBLE HYPOTHESES		REJECTION REGION	REJECT IF
TWO-SIDED	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$		$z < -z_{\alpha/2}$ OR $z > z_{\alpha/2}$
ONE-SIDED	$H_0: \mu = \mu_0$ $H_a: \mu < \mu_0$		$z < -z_\alpha$
ONE-SIDED	$H_0: \mu = \mu_0$ $H_a: \mu > \mu_0$		$z > z_\alpha$



TYPICAL VALUES FOR α, THE <u>SIGNIFICANCE LEVEL</u> OF THE TEST,

$\alpha = .10$
$\quad = .05$
$\quad = .01$

WE CAN MAKE TWO TYPES OF ERRORS

TYPE I ERROR, REJECT H_0 WHEN H_0 IS TRUE

TYPE II ERROR, FAIL TO REJECT H_0 WHEN H_0 IS FALSE.

	STATE OF NATURE	
DECISION	H_0 TRUE	H_0 FALSE
FAIL TO REJECT H_0	CORRECT DECISION	TYPE II ERROR
REJECT H_0	TYPE I ERROR	CORRECT DECISION

TYPICALLY FIX PROBABILITY OF MAKING A TYPE I ERROR = α.

3/3 In Example 1, we rejected H_0 at the $\alpha = .05$ level with $z_{\alpha/2} = 1.96$. We were willing to accept a 5% chance of rejecting H_0 if H_0 is true.

The probability of making a Type II error $= \beta$,
the probability of failing to reject H_0 when H_0 is false.

The <u>Power of the test</u> $= 1 - \beta$
the probability of rejecting H_0 when it is false.

The power is computed for specific values of the parameter in the alternative hypothesis.

EX Find the power $(1-\beta)$ of the hypothesis test when the true value of $\mu = 1.01$ in.

$\beta = Pr(\text{Do not reject } H_0: \mu = 1.00 \text{ when } H_0 \text{ is false } (\mu = 1.01))$

Let $\alpha = 0.05$.

Assume H_0 is true, then $\bar{X} \sim N(1.00, \sigma^2/n)$.

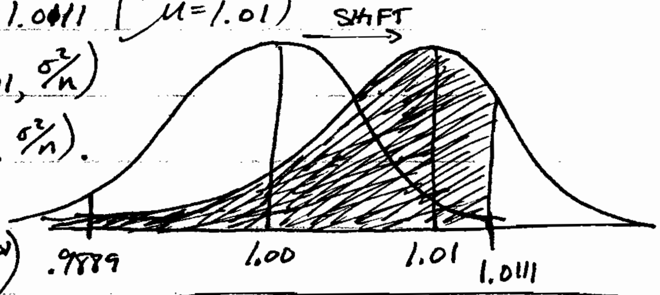
A 95% CI is $(.9889 \le \bar{X} \le 1.0111)$.

We do not reject when $\bar{X}$ is in this interval.

We want $\beta = Pr(\text{Do not reject} \mid H_0 \text{ is false } (\mu = 1.01))$

$= Pr(.9889 \le \bar{X} \le 1.0111 \mid \mu = 1.01)$

It is the area under $N(1.01, \sigma^2/n)$ inside the 95% CI of $N(1, \sigma^2/n)$.



$$Pr\left(\frac{.9889 - 1.01}{.04/\sqrt{50}} \le \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \le \frac{1.0111 - 1.01}{.04/\sqrt{50}}\right)$$

$= Pr(-3.73 \le Z \le 0.194)$

$= .575345 - .000096 = .575$

So Power $= 1 - \beta = 1 - .575 = .425$

Power is used to select sample size needed to detect a given difference. Typically, increase sample size to increase power.

If the mean is actually $\mu = 1.01$, then this test will correctly reject H_0 only 42.5% of the time.

7.2 ½ <u>P-VALUES</u> GIVE THE OBSERVED SIGNIFICANCE LEVEL OF THE TEST.

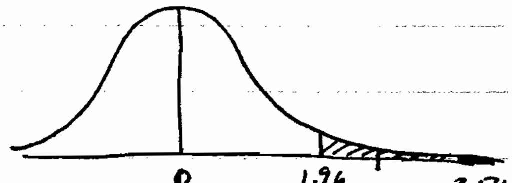
RECALL EX 1

$H_0: \mu = 1.00$ vs $H_a: \mu \neq 1.00$

$n = 50$, $\bar{X} = 1.02$, $s = .04$, ~~[illegible]~~

$\alpha = .05 \rightarrow$ REJECT IF $Z < -1.96$ OR $Z > 1.96$

TEST STATISTIC IS $Z = \dfrac{\bar{X} - 1}{s/\sqrt{n}} = 3.54.$



IF $\alpha = .01$ LEVEL TEST, $Z_{\frac{\alpha}{2}} = 2.576$

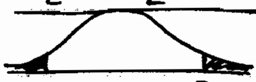
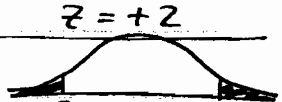
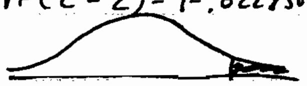
$Z = 3.54 > Z_{\frac{.01}{2}} = 2.576 \Rightarrow$ REJECT H_0 AT $\alpha = .01$

P-VALUE GIVES <u>SMALLEST α-VALUE</u> CAN REJECT TEST AT.

$$\text{P-VALUE} = P_r(Z \leq -3.54) + P_r(Z \geq 3.54) = 2\,P_r(Z < -3.54) = 2(.0002) = .0004$$

P-VALUE IS THE PROBABILITY GIVEN H_0 IS TRUE THAT WE OBSERVE A VALUE <u>AT LEAST AS EXTREME</u> AS WE DID IN THE DIRECTION OF H_a.

<u>EX</u>

<u>ALTERNATIVE</u>	<u>P-VALUE WHEN</u> $Z = -2$	$Z = +2$
$H_a: \mu \neq \mu_0$	 $z=-2$ 2 $2P_r(Z \leq -2) = 2(.022750) = .0455$	 -2 $z=2$ $2P_r(Z \geq 2) = .0455$
$H_a: \mu < \mu_0$	 $z=-2$ $P_r(Z \leq -2) = .022750$	 $z=2$ $P_r(Z \leq 2) = 1 - .022750 = .97725$
$H_a: \mu > \mu_0$	 $z=-2$ $P_r(Z \geq -2) = .97725$	 $z=2$ $P_r(Z \geq 2) = .022750$

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THE FOUR STEPS TO HYPOTHESIS TESTING

1. STATE HYPOTHESIS, BOTH IN WORDS (IN TERMS OF THE ALTERNATIVE) AND IN THE FOLLOWING NOTATION:

   $H_0: \mu = \mu_0$

   $H_a: \mu \neq \mu_0$ OR $\mu < \mu_0$ OR $\mu > \mu_0$.

2. CALCULATE TEST STATISTIC

NORMAL POPULATION W/ KNOWN VARIANCE OR LARGE SAMPLE ($n > 40$)	SMALL SAMPLE, UNKNOWN VARIANCE, ROUGHLY NORMAL	POPULATION PROPORTION LARGE SAMPLE ($np_0 > 5$, $n(1-p_0) > 5$)
$Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$	$T = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$	$Z = \dfrac{X - np_0}{\sqrt{np_0(1-p_0)}} = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$

3. FIND P-VALUE ASSOCIATED WITH TEST STATISTIC IN THE DIRECTION OF H_a.
   [SEE P-VALUE ABOVE]

4. COMPARE P-VALUE TO α-LEVEL. IF P-VALUE $< \alpha$, REJECT H_0.
   IF P-VALUE $\geq \alpha$, FAIL TO REJECT H_0.

   WHEN POSSIBLE STATE CONCLUSION IN TERMS OF THE ORIGINAL PROBLEM.

5. CHECK ASSUMPTIONS OF TEST: NORMALITY = ROUGHLY SYMMETRIC, UNIMODAL, NO OUTLIERS

[HANDOUT 10]

EX 2: LARGE SAMPLE Z $H_a: \mu \neq 3500$

EX 3: LARGE SAMPLE Z $H_a: \mu > 0$

EX 4: LARGE SAMPLE Z $H_a: \mu < 1000$

9.3.5 $\frac{1}{2}$ OUR THREE HYPOTHESIS TESTS.

Z. TEST OF POPULATION MEAN,

NORMAL POPULATION W/ KNOWN VARIANCE OR

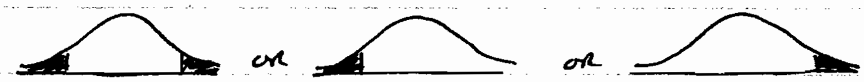
LARGE SAMPLE ($n \geq 40$)

1. $H_0: \mu = \mu_0$

   $H_a: \mu \neq \mu_0$ OR $\mu < \mu_0$ OR $\mu > \mu_0$

2. $Z = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$

3. P-VAL FROM TABLE II



4. CONCLUSION

T. TEST FOR POPULATION MEAN,

SMALL SAMPLE ($n < 40$) W/ UNKNOWN VARIANCE, ROUGHLY NORMAL

1. SAME AS (Z)
2. $T = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$

3. $df = n - 1$,

   P-VAL BOUNDS FROM TABLE IV

   FIND VALUES IN TABLE ON df ROW BORDERING T VALUE.

   P-VALUE IS BETWEEN PROBABILITY BOUNDS AT TOP OF TABLE.

   (FOR TWO TAILED TEST $H_a: \mu \neq \mu_0$, PROBABILITY BOUNDS ARE DOUBLED).

4. CONCLUSION.

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P. TEST OF POPULATION PROPORTION

LARGE POPULATION, SAMPLE LARGE ENOUGH SO $np_0 > 5$ AND $n(1-p_0) > 5$.

1. $H_0: p = p_0$

   $H_a: p \neq p_0$ OR $p < p_0$ OR $p > p_0$

2. $Z = \dfrac{X - np_0}{\sqrt{np_0(1-p_0)}} = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$, $\hat{p} = \dfrac{X}{n}$

3. SAME AS (Z)

4. CONCLUSION

[HANDOUT 11]

Ex1: T $H_a: \mu \neq 224$

Ex2: T $H_a: \mu > 0$

Ex3: P $H_a: p \neq .73$