Course Notes 2/2

STAT 345 Elements of Math. Stats and Prob Theory
UNM

Erik Erhardt

Covered	Pages	Materials	Quiz	Exam
4.2	2			
4.3	1		8	
4.4	3			
4.5	2			
4.9	1		9	
4.6	3	Handout 4,5		
4.7	1			2
5.1	2			
5.5	4			
5.7	4	Handout 6		
Exam 2	1	Take home to prep for		
6.1.2	2			
6.4.3.5	1			
7.4	1	Handout 6 (6's results)		
7.5	1	Handout 6b		
8.1.2	5	Handout 7		3
8.3	3	Handout 8		
8.5	1	Handout 9		
9.1	3	Handout 10		
9.2	2	Handout 10		
9.3.5	2	Handout 11		
Exam 3				

4.2 ½

4.2 CONTINUOUS RVs AND PROBABILITY DISTRIBUTIONS

(4.1) RECALL THAT A CONTINUOUS RV CAN TAKE ON AN INTERVAL OF NUMBERS.

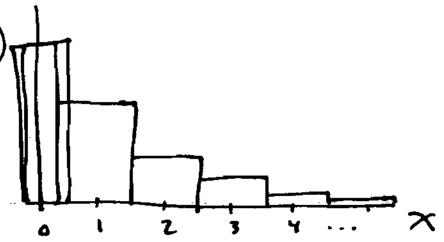
EX MEASURE THE LIFETIME OF AN ELECTRONIC COMPONENT,

X = LIFETIME IN HOURS

THE RANGE OF X IS $\{x : 0 \le x < \infty\}$

CONSIDER A DISCRETE RV X WITH RANGE $\{0, 1, 2, \ldots\}$.

SUPPOSE PMF OF X IS

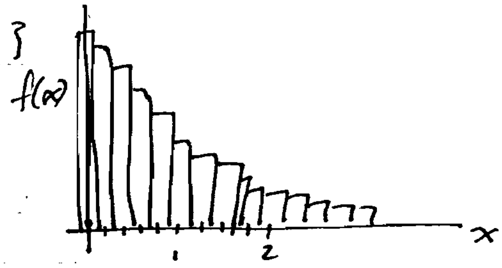


CDF OF X IS $F(x) = P(X \le x) = \sum_{x_i \le x} f(x_i)$

COMPUTE AREAS OF EACH RECTANGLE UNTIL THE ONE YOU WANT.

AREAS SUM TO GIVE $P(X \le x) = F(x)$.

TAKE RANGE TO BE $\{0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \ldots\}$



CONTINUE TO REDUCE WIDTH OF RECTANGLES.

INFINITE SUM OF RECTANGLES = INTEGRAL.

TAKE SMALLER AND SMALLER VALUES IN THE RANGE SPACE,

$$\sum_{x_i < x} f(x) = \int_0^x f(u)\,du.$$

FOR CONTINUOUS RV, THE HISTOGRAM APPROXIMATES $f(x)$.

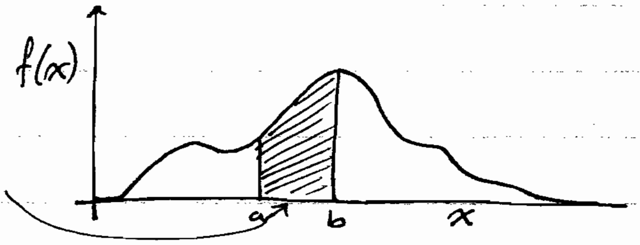
$f(x)$ IS CALLED A <u>PROBABILITY DENSITY FUNCTION (pdf)</u> FOR CONTINUOUS

(PMF FOR DISCRETE)

2/2

PDF OF A CONT. RV.

$Pr(a \le x \le b)$ = AREA UNDER CURVE



PROPERTIES OF THE PDF, $f(x)$:

1. $f(x) \ge 0$
2. $\int_{-\infty}^{\infty} f(x)dx = 1$
3. $Pr(a \le x \le b) = \int_a^b f(x)dx$ = AREA UNDER $f(x)$ FROM a TO b.

EX $Pr(x=68) = 0$

IF ↑ ROUNDED VALUE OF A CONTINUOUS R.V.,

$Pr(67.5 \le x < 68.5)$ MAKES MORE SENSE

NOTE: $Pr(a \le x \le b) = Pr(a < x \le b) = Pr(a \le x < b) = Pr(a < x < b) = \int_a^b f(x)dx$

EX

SUPPOSE X IS THE LIFETIME OF AN ELECTRICAL COMPONENT IN YEARS AND THE PDF OF X IS $f(x) = \frac{1}{4}e^{-\frac{1}{4}x}$, $x \ge 0$.

a) VERIFY $f(x)$ IS A PDF

b) FIND PROBABILITY THAT A RANDOMLY SELECTED COMPONENT LASTS LONGER THAN 3 YRS.

c) FIND PROBABILITY IT LASTS BETWEEN 1 AND 2 YEARS.

a) $\int_{-\infty}^{\infty} f(x)dx = \int_0^{\infty} \frac{1}{4}e^{-\frac{1}{4}x}dx = \frac{1}{4}\int_0^{\infty} e^{-\frac{1}{4}x}dx$ $\quad u = -\frac{1}{4}x$

$du = -\frac{1}{4}dx \Rightarrow dx = -4du$

$= \frac{1}{4}\int e^u(-4du) = -\int e^u du = -e^u = -e^{-\frac{1}{4}x}\Big|_0^{\infty}$

$= \lim_{x\to\infty} -e^{-\frac{1}{4}x} - (-e^0) = 0 + 1 = 1$ ✓

b) $Pr(X>3) = \int_3^{\infty} \frac{1}{4}e^{-\frac{1}{4}x}dx = -e^{-\frac{1}{4}x}\Big|_3^{\infty} = 0 - (-e^{-\frac{3}{4}}) = .4724$

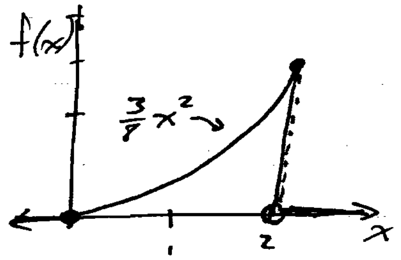
or $= 1 - Pr(X \le 3) = 1 - \int_0^3 \frac{1}{4}e^{-\frac{1}{4}x}dx$

c) $Pr(1 \le x \le 2) = \int_1^2 \frac{1}{4}e^{-\frac{1}{4}x}dx = -e^{-\frac{1}{4}x}\Big|_1^2 = -e^{-2/4} - (-e^{-1/4}) = .1723$

4.3 $1

4.3 CDFs.

EX THE PDF FOR A R.V. X IS GIVEN BY
$f(x) = cx^2, \quad 0 \le x \le 2$



a) FIND c SO THAT $f(x)$ IS A PDF

b) FIND $Pr(X \le 3)$

c) FIND THE CDF OF $f(x)$

a) $\int_0^2 c x^2 \, dx = 1 = c \frac{x^3}{3}\Big|_0^2 = \frac{c}{3}(2^3 - 0) = \frac{8}{3} c = 1$

$\Rightarrow c = \frac{3}{8}.$

b) $Pr(X \le 3) = 1$, ALL!

c) $F(x) = Pr(X \le x)$ CDF'S ARE DEFINED ON $(-\infty, \infty)$.

$$F(x) = \begin{cases} 0, & x < 0 \\ \int_0^x \frac{3}{8} t^2 \, dt, & 0 \le x \le 2 \\ 1, & x > 2 \end{cases}$$

(REPLACE w/ t SO NOT SO MANY x's)

$$\int_0^x \frac{3}{8} t^2 \, dt = \frac{3}{8} \frac{t^3}{3}\Big|_0^x = \frac{x^3}{8}$$

$$F(x) = \begin{cases} 0, & x < 0 \\ x^3/8, & 0 \le x \le 2 \\ 1, & x > 2 \end{cases}$$

EX SUPPOSE R.V. X HAS pdf $f(x) = 6(x - x^2), \quad 0 \le x \le 1$

a) SHOW $\int_{-\infty}^{\infty} f(x) \, dx = 1$.

b) FIND $Pr(X > \frac{1}{2})$

c) FIND cdf.

a) $\int_0^1 6(x - x^2) \, dx = 6\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1 = 6\left(\frac{1}{2} - \frac{1}{3} - (0 - 0)\right) = 6\left(\frac{1}{6}\right) = 1$

b) $Pr(X > \frac{1}{2}) = \int_{1/2}^{1} 6(x - x^2) \, dx = 6\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{1/2}^{1} = 6\left(\frac{1}{2} - \frac{1}{3} - \left(\frac{1}{8} - \frac{1}{24}\right)\right)$
$= 6\left(\frac{1}{6} - \frac{1}{12}\right) = \frac{1}{2}.$

c) $F(x) = Pr(X \le x) = \begin{cases} 0, & x < 0 \\ \int_0^x 6(t - t^2)\,dt, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}$

$\rightarrow \frac{6t^2}{2} - \frac{6t^3}{3}\Big|_0^x = 3x^2 - 2x^3$

4.4 1/3

4.4 Mean and Variance of Continuous RV.

$$E(X)=\int_{-\infty}^{\infty} x f(x)\,dx\ ,\quad E(g(x))=\int_{-\infty}^{\infty} g(x)f(x)\,dx\ ,\quad Var(X)=E(X^2)-(E(x))^2 = \int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx$$

Ex Suppose X has pdf $f(x)=ke^{-3x}$, $x\geq 0$.

a) Find k

b) Find $F(x)$

c) Find $E(x)$ and $Var(x)$.

a) $\int_0^{\infty} ke^{-3x}\,dx = 1 = \frac{k}{(-3)}\int_0^{\infty} e^{u}\,du = -\frac{k}{3}e^{u} = -\frac{k}{3}e^{-3x}\Big|_0^{\infty}$

$u=-3x$

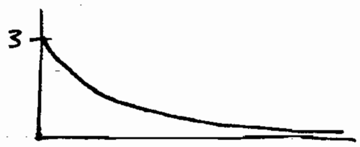
$du=-3dx$

$dx=\frac{du}{-3}$

$= -\frac{k}{3}(0-1) = \frac{k}{3} = 1$

$\Rightarrow k=3.$

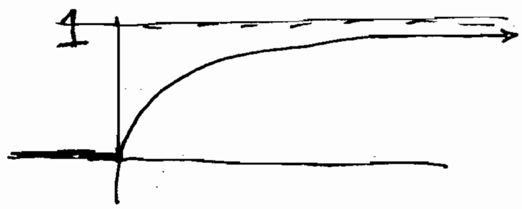
b) $f(x)=3e^{-3x}$, $x\geq 0$.



$$F(x)=\begin{cases} 0 & x<0 \\ \int_0^{x} 3e^{-3t}\,dt & x\geq 0\end{cases}$$

$$\int_0^x 3e^{-3t}\,dt = \frac{3}{-3}e^{-3t}\Big|_0^x = -1\left(e^{-3x}-1\right) = 1-e^{-3x}$$

$$F(x)=\begin{cases} 0 & x<0 \\ 1-e^{-3x} & x\geq 0\end{cases}$$



c) $E(X)=\int_{-\infty}^{\infty} x f(x)\,dx$, $Var(x)=E(X^2)-(E(x))^2$

$E(X)=\int_0^{\infty} x\,3e^{-3x}\,dx$

By parts: $\int u\,dv = uv-\int v\,du$ * Next Page →

$u=3x$ $dv=e^{-3x}dx$

$du=3dx$ $v=-\frac{1}{3}e^{-3x}$

$E(X)=-\frac{3x}{3}e^{-3x}-$

2/3

c) $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$ $\qquad$ $\mathrm{VAR}(X) = E(X^2) - (E(X))^2$

$$E(X) = \int_0^{\infty} 3x e^{-3x}\,dx = \frac{3x}{-3} e^{-3x}\Big|_0^{\infty} - \frac{3}{9} e^{-3x}\Big|_0^{\infty}$$

BY PARTS

u		dv
$3x$	$+$	e^{-3x}
3	$-$	$-\frac{1}{3}e^{-3x}$
0		$+\frac{1}{9}e^{-3x}$

WORKS IF ENDS WITH ZERO

$$= (0-0) - \tfrac{1}{3}(0-1)$$

$$= \tfrac{1}{3}.$$

$E(X) = \frac{1}{3} = \mu_{MEAN}$.

$$E(X^2) = \int_0^{\infty} 3x^2 e^{-3x}\,dx = \frac{3x^2}{-3} e^{-3x}\Big|_0^{\infty} - \frac{6x}{9} e^{-3x}\Big|_0^{\infty} + \frac{6}{-27} e^{-3x}\Big|_0^{\infty}$$

u		dv
$3x^2$	$+$	e^{-3x}
$6x$	$-$	$-\frac{1}{3}e^{-3x}$
6	$+$	$\frac{1}{9}e^{-3x}$
0		$-\frac{1}{27}e^{-3x}$

$$= (0-0) - (0-0) + \tfrac{6}{-27}(0-1)$$

$$= \tfrac{6}{27} = \tfrac{2}{9}$$

$E(X^2) = \frac{2}{9}$

$$\mathrm{VAR}(X) = E(X^2) - (E(X))^2 = \tfrac{2}{9} - \left(\tfrac{1}{3}\right)^2 = \tfrac{2}{9} - \tfrac{1}{9} = \tfrac{1}{9}$$

$\mathrm{VAR}(X) = \frac{1}{9}$.

PROPERTIES OF $E(X)$ AND $\mathrm{VAR}(X)$

IF c IS A CONSTANT,

$E(cX) = c\,E(X)$

$E(X+c) = E(X) + c$

$\mathrm{VAR}(cX) = c^2\,\mathrm{VAR}(X)$

$\mathrm{VAR}(X+c) = \mathrm{VAR}(X)$.

3/3

EX SUPPOSE THAT THE CDF OF A RV X IS GIVEN BY

$$F(x) = \begin{cases} 0, & x \le 0 \\ x^3, & 0 \le x \le 1 \\ 1, & 1 < x \end{cases}$$

a) FIND pdf $f(x)$

b) FIND $E(e^x)$

a) $$f(x) = \frac{d}{dx}F(x) = \begin{cases} \frac{d}{dx}0, & x < 0 \\ \frac{d}{dx}x^3, & 0 \le x \le 1 \\ \frac{d}{dx}1, & 1 < x \end{cases}$$

$$= \begin{cases} 0, & x < 0 \\ 3x^2, & 0 \le x \le 1 \\ 0, & 1 < x \end{cases}$$

$f(x) = 3x^2, \quad 0 \le x \le 1.$

b) $$E(e^x) = \int_0^1 e^x(3x^2)\,dx = 3x^2e^x\Big|_0^1 - 6xe^x\Big|_0^1 + 6e^x\Big|_0^1$$

$$= (3e^1 - 0) - 6(e^1 - 0) + 6(e^1 - 1)$$

$$= 3e^1 - 6 \doteq 2.155$$

BY PARTS

u		dv
$3x^2$	+	e^x
$6x$	−	e^x
6	+	e^x
0		e^x

4.5 $\frac{1}{2}$

4.5 Continuous Uniform Distribution

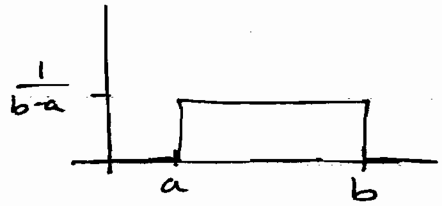
X has a continuous uniform distribution if every value in an interval has the same ~~[illegible]~~ likelihood.

$$X \sim U(a, b)$$

$$f(x) = \frac{1}{b-a}, \quad a \le x \le b$$

$$E(X) = \frac{a+b}{2}$$

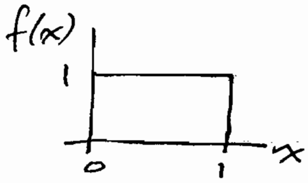
$$\mathrm{Var}(X) = \frac{(b-a)^2}{12}$$



EX Suppose you want to randomly select a number represented by a point in the interval $0 \le x \le 1$.

X = number between 0 and 1

$X \sim U(0,1)$, $\quad f(x) = 1, \quad 0 \le x \le 1.$



Used to model things like:

- Equally likely waiting times
- Beam loadings
- Dart board example

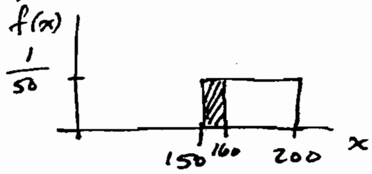
EX The thickness of steel sheets from rolling machines have a uniform distribution with values between 150 mm – 200 mm. Any sheets with thickness less than 160 mm are scrapped.

a) Calculate the fraction scrapped.

b) Find the mean and std dev of thickness.

X = thickness in mm of steel sheet

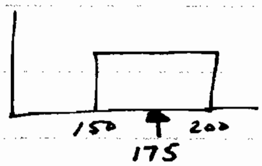
$X \sim U(150, 200)$, $\quad f(x) = \frac{1}{200-150} = \frac{1}{50}, \quad 150 \le x \le 200$



2/2

a) $Pr(X<160) = \int_{150}^{160} \frac{1}{50}\,dx = \frac{1}{50}x\Big|_{150}^{160} = \frac{1}{50}(160-150) = \frac{10}{50} = .2$

$= 20\%$



b) $\mu = 175$ BALANCE-POINT.

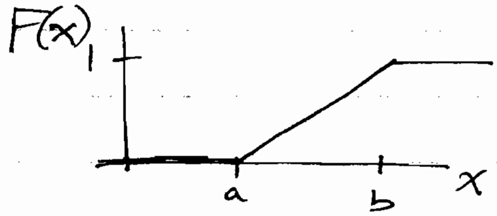
$$E(X) = \int_{150}^{200} x\,\frac{1}{50}\,dx = \frac{x^2}{2(50)}\Big|_{150}^{200} = \frac{200^2-150^2}{2(200-150)} = \frac{200+150}{2}$$

$$= \frac{b^2-a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} = \frac{b+a}{2} \checkmark = 175$$

$$E(X^2) = \int_{150}^{200} x^2\,\frac{1}{50}\,dx = \frac{1}{50}\frac{x^3}{3}\Big|_{150}^{200} = \frac{1}{150}(200^3-150^3) = 30833\tfrac{1}{3}$$

$$VAR(X) = E(X^2) - (E(X))^2 = 30833\tfrac{1}{3} - 175^2 = 208\tfrac{1}{3}$$

$$STD(X) = \sqrt{\sigma^2} = \sqrt{208\tfrac{1}{3}} = 14.43\,mm$$



CDF

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \begin{cases} 0, & x<a \\ \int_a^x \frac{1}{b-a}\,dt, & a\le x\le b \\ 1, & x>b \end{cases}$$

$$= \begin{cases} 0, & x<a \\ \frac{x-a}{b-a}, & a\le x\le b \\ 1, & x>b \end{cases}$$

4.9 1/1

4.9 Exponential Distribution

Recall Poisson RV counts number of successes over an interval of time or area or, etc.

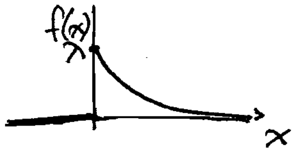
~~Th~~ ~~then~~ X has an exponential distribution if it measures the distance between successive counts in a Poisson process.

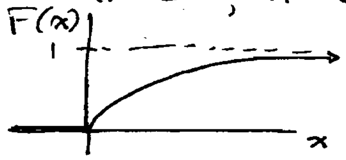
$X \sim Exp(\lambda)$

$f(x) = \lambda e^{-\lambda x}, \quad 0 \le x < \infty, \lambda > 0$ $\quad F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\lambda x}, & x \ge 0 \end{cases}$

$E(X) = \frac{1}{\lambda}$

$Var(X) = 1/\lambda^2$





<u>Ex</u> Suppose the number of arrivals at a service counter follows a Poisson distribution with mean number of arrivals $\lambda = 3$ per hour. Then the length of time between 2 successive arrivals, X, has $X \sim Exp(3)$, $f(x) = 3e^{-3x}$, $x \ge 0$.

$E(X) = \frac{1}{3}$, $Var(X) = \frac{1}{9}$, $\sigma = \sqrt{1/9} = \frac{1}{3}$.

(1st <u>Ex</u> in 4.4)

<u>Ex</u> The time until failure, in hours, of a fan in a computer can be modelled as $Exp(0.0003)$.

a) What proportion of fans will last at least 10,000 hours?

b) What proportion of fans will last at most 7,000 hours?

c) What is a fan's expected lifetime?

X = time until failure in hours

$X \sim Exp(0.0003)$

a) $Pr(X \ge 10{,}000) = \int_{10000}^{\infty} 0.0003\, e^{-0.0003x}\, dx = -e^{-0.0003x}\Big|_{10000}^{\infty}$

$= 0 - (-e^{-3}) = e^{-3} = .0498 \approx .05$

b) $Pr(X \le 7000) = F(7000) = 1 - e^{-0.0003(7000)} = 1 - e^{-2.1} = .8775 \approx .88$

c) $E(X) = \frac{1}{0.0003} = 3333\tfrac{1}{3}$ hours

4.6 1/3

4.6 NORMAL DISTRIBUTION

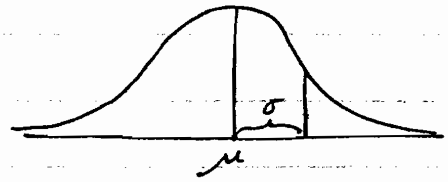
X HAS A NORMAL DISTRIBUTION IF IT MEASURES A PROCESS THAT HAS A CENTER AND IS SUBJECT TO ERROR OR SYMMETRIC DEVIATIONS, FOR EXAMPLE. ARISES IN MANY OTHER SITUATIONS.

$$X \sim N(\mu, \sigma^2), \quad -\infty < \mu < \infty, \quad \sigma^2 > 0.$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, \quad -\infty < x < \infty$$

$$E(X) = \mu$$

$$\text{VAR}(X) = \sigma^2$$



NOTE: $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$ STANDARD NORMAL.

$\Phi(z) = Pr(Z \le z) =$ LOOK UP IN TABLE II, PP. 653-4.

EX $Z \sim N(0,1)$, MEAN $= \mu = 0$, VARIANCE $= \sigma^2 = 1$.

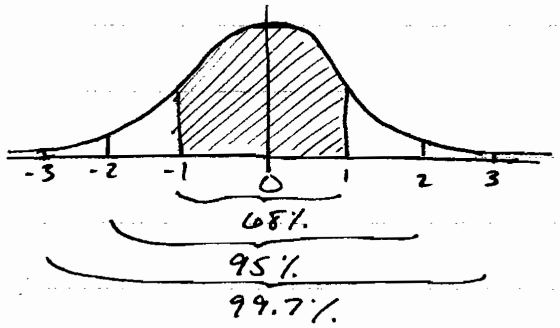
$$f(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(z-\mu)^2} = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \quad -\infty \le z < \infty$$

68-95-99.7 RULE

$Pr(-1<Z<1) = .6827$ (1 STD)

$Pr(-2<Z<2) = .9545$ (2 STD)

$Pr(-3<Z<3) = .9973$ (3 STD)



EX IF Z HAS A STANDARD NORMAL DISTRIBUTION, FIND

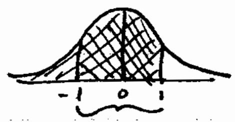
a) $Pr(Z<1)$

b) $Pr(Z<-2)$

c) $Pr(Z>2)$

d) $Pr(Z<0)$

a)



KNOW .6827

$\Rightarrow$ TAILS $= 1 - .6827 = .3173$

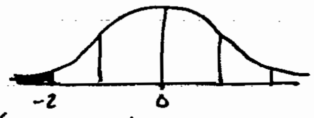
LEFT TAIL

BY SYMMETRY, $Pr(Z<1) = .6827 + \frac{1}{2}(.3173)$

$= .6827 + .1586$

$= .84$

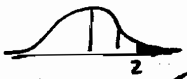
b)



$Pr(-2<Z<2) = .9545$

$Pr(Z<-2) = \frac{1-.9545}{2} = .0228$

c) SAME AS (b)



d)

$= \frac{1}{2}$.

2/3

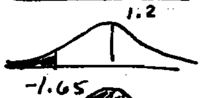
USING TABLE II pp. 653-4.
FIND a) $Pr(Z \le 0.48)$
b) $Pr(Z > 1.2)$
c) $Pr(Z < -1.65)$
d) $Pr(-1.65 \le Z \le 0.48)$.

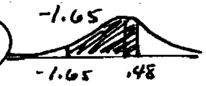
MOTIVATION:
$$\star\ Pr(Z \le 3) = \int_{-\infty}^{3} f(t)\,dt$$
$$= \int_{-\infty}^{3} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}\,dt$$

NO ANTIDERIVATIVE!
LET THE COMPUTER DO IT,
$\Rightarrow$ USE TABLES. [REFER HANDOUT 5 / COVERED AT END OF SECTION.]

a) $= 0.684386$


b) $= 1 - 0.884930 = .1151$

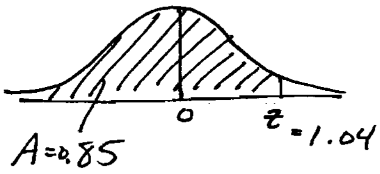

c) $= 0.049471$


d) $= Pr(Z \le 0.48) - Pr(Z < -1.65)$


$= 0.684386 - 0.049471$
$= 0.634915$

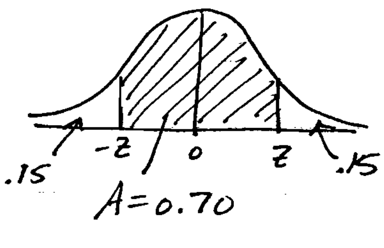
EX IF $Z \sim N(0,1)$, FIND z SUCH THAT $Pr(Z \le z) = .85$

LOOK IN BODY OF TABLE FOR AREA = .85
CHOOSE CLOSEST Z-SCORE (VALUE OF z).
$Pr(Z \le 1.04) = 0.850830$

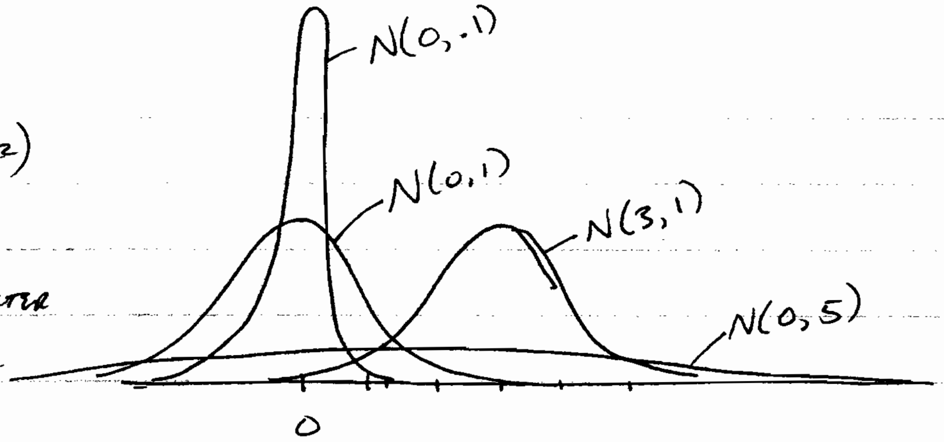


EX FIND z SUCH THAT $Pr(-z \le Z \le z) = 0.70$.

CONVERT TO AN "AREA TO THE LEFT"
$Pr(Z \le z) = .7 + .15 = .85$
(SAME AS ABOVE)



3/3 $X \sim N(\mu, \sigma^2)$



μ = LOCATION PARAMETER

σ^2 = SCALE PARAMETER

$$Z = \frac{X-\mu}{\sigma} \sim N(0,1)$$

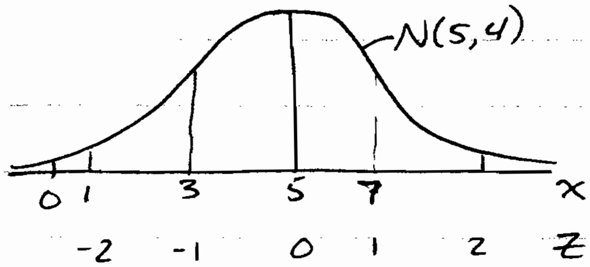
EX Suppose $X \sim N(5,4)$, $\mu=5$, $\sigma^2=4$ so $\sigma=2$

FIND

a) ~~FIND~~ $P_r(X \le 7)$

b) ~~FIND~~ $P_r(3 \le X \le 7)$

c) ~~FIND~~ $P_r(X \le 7.3)$



CONVERT TO Z AND USE TABLE.

a) $P_r(X \le 7)$

$= P_r\left(\frac{X-\mu}{\sigma} \le \frac{7-5}{2}\right)$

$= P_r(Z \le 1) = 0.8413$

b) $P_r(3 \le X \le 7) = P_r\left(\frac{3-5}{2} \le \frac{X-\mu}{\sigma} \le \frac{7-5}{2}\right) = P_r(-1 \le Z \le 1) = .68$

c) $P_r(X \le 7.3) = P_r\left(\frac{X-\mu}{\sigma} \le \frac{7.3-5}{2}\right) = P_r(Z \le 1.15) = 0.8749$.

HANDOUT 4]

HANDOUT 5]

4.7 1/1

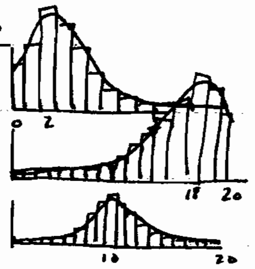
4.7 Normal Approximation to the Binomial and Poisson Distributions

<u>Binomial Approximation</u>

Bin(20, .1)

Bin(20, .9)

Bin(20, .5)



} Not Normal

– Looks Normal,

$X \sim Bin(20, .5)$ can be approximated by a Normal Distribution.

X = number of successes out of $n = 20$ trials, $p = .5$

$\mu = E(X) = np$

$\sigma^2 = Var(X) = np(1-p)$

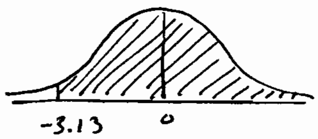
$$Z = \frac{X-\mu}{\sigma} = \frac{X-np}{\sqrt{np(1-p)}} = \frac{X-20(.5)}{\sqrt{20(.5)(1-.5)}} \overset{a}{\sim} N(0,1)$$

Now can use Normal Dist. Probabilities for Binomial Dist.

<u>EX</u> if $X \sim Bin(20, .5)$, estimate $Pr(X \geq 3)$ using Normal Distribution.

$$Pr(X \geq 3) = Pr\left(\frac{X-\mu}{\sigma} \geq \frac{3-20(.5)}{\sqrt{20(.5)(.5)}}\right) \approx Pr\left(Z \geq \frac{3-10}{\sqrt{5}}\right) = Pr(Z \geq -3.13)$$

$= .99913$

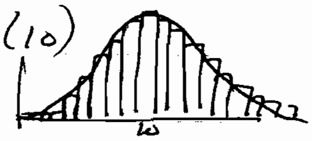


The approximation is good for if

Both $np > 5$ and $n(1-p) > 5$.

<u>Poisson Approximation</u>

Same as Binomial, but easier. Note $X \sim Pois(10)$



$\mu = \lambda$, $\sigma^2 = \lambda$.

if $X \sim Pois(\lambda)$, then $Z = \frac{X-\lambda}{\sqrt{\lambda}} \overset{a}{\sim} N(0,1)$ approximately.

Good when $\lambda > 5$.

For both approximations, much easier to get probabilities from Table II than ~~adding up~~ taking (often large) summations