Course Notes 1

Stat 345 Elements of Math. Stats and Prob. Theory

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Covered	Pages	Materials	Quiz	Exam
Intro	2	Handout 1,2	1	
2.1	4		2	
2.2	2			
2.3	2		3	
2.4	2			
2.5	2			
2.6	2			
2.7	1			
Quiz 3 Review	1		Prep 3	
2.1 cd	2			
2.8	1			
3.1	1		4	
3.2	3			
3.3	2			
3.4	3			
Exam 1 topics	½			1
3.5	2			
3.6	3		5, 6,8 7	
3.7	4	Handout 3		
3.8	1			
3.9	3			
Calc Review	1			

Continued in Course Notes 2

INTRO 1/2

INTRODUCTION : ELEMENTS OF MATHEMATICAL STATISTICS & PROBABILITY THEORY.

PROBABILITY

WE MAKE CONCLUSIONS ABOUT A SAMPLE BASED ON OUR KNOWLEDGE OF THE POPULATION.

STATISTICS

WE MAKE CONCLUSIONS ABOUT THE POPULATION BASED ON OUR KNOWLEDGE OF ~~THE~~ A SAMPLE.

A POPULATION CONSISTS OF THE TOTALITY OF THE OBSERVATIONS WITH WHICH WE ARE CONCERNED.

A SAMPLE IS A SUBSET OF OBSERVATIONS SELECTED FROM A POPULATION.

[HANDOUT 1] LEXINGTON LEFT-TURNS.

EX 1: POPULATION : ALL ACCIDENTS AT INTERSECTIONS IN LEXINGTON, KY.

SAMPLE : ACCIDENTS IN 5 YEAR PERIOD AT ~~#~~ A NUMBER OF INTERSECTIONS IN STUDY, SOME WITH AND WITHOUT LEFT-TURN LANES.

EX 2 : VEGETABLE COOLING

POPULATION : ALL GREEN TOMATO PLANTS

SAMPLE : 26 BATCHES OF TOMATOES.

EX 3 : DDT IN FISH

POPULATION : ALL FISH DOWN STREAM OF PLANT IN TENN. RIVER

SAMPLE : 144 FISH

SAMPLE STATISTICS

CENTERS

MEAN $\bar{x} = \frac{1}{n}(x_1 + x_2 + \cdots + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i$ $\qquad \bar{x} = 24.35$ ppm

MEDIAN M = MIDDLE OBSERVATION OF SORTED DATA $\qquad M = 7.15$ ppm

TR MEAN = TRUE MEAN = ELIMINATE OUTLIERS.

2/2

Spread

StDev: $s = \sqrt{\frac{1}{n-1} \sum (X_i - \bar{X})^2}$, $s = \sqrt{\frac{1}{143} \left[(.11 - 24.35)^2 + \cdots + (1100 - 24.35)^2 \right]}$ $s = 98.38$ ppm

Variance is s^2

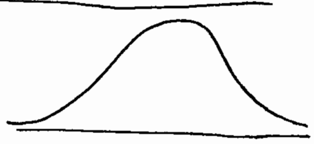
Quantiles – Five-Number Summary

min	Q_1	M	Q_3	max
.11	3.33	7.15	13.00	1100.00
Minimum	First Quartile	Median	Third Quartile	Maximum

Range = max − min = 1099.89

Interquartile Range = $Q_3 - Q_1$ = 9.67

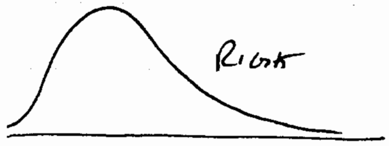
Graphical Summaries



Symmetric

Unimodal

Bell-Shaped
(Roughly Normal)



Right-Skewed

Unimodal

Left-Skewed

Unimodal

Bimodal, Slight Right-Skewed.

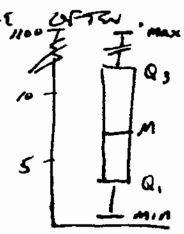
[Handout 1] **Histograms**

Frequency Plot of Observed Data

DDT: Smaller values occur more often.

Box Plot

Plot of Five-number summary



2.1 1/4

CH 2 PROBABILITY

2.1 SAMPLE SPACES AND EVENTS

AN EXPERIMENT IS ANY PROCESS OF OBTAINING AN OBSERVATION (DATA).

EX. 1: COUNT NUMBER OF ACCIDENTS AT AN INTERSECTION IN LEXINGTON, KY.

EX 3: MEASURE DDT CONCENTRATION IN FISH IN TENN RIVER.

RANDOM EXPERIMENT: IF REPEATED IN THE MANNER EVERY TIME, GET DIFFERENT OUTCOMES.

SAMPLE SPACE, S: ALL POSSIBLE OUTCOMES OF A RANDOM EXPERIMENT.

EX. 1: $S_1 = \{0, 1, 2, 3, \ldots\}$ ALL POSSIBLE COUNTS OF ACCIDENTS

EX 3: $S_3 = \{x : 0 \le x < \infty\}$ ANY POSITIVE REAL NUMBER (DDT)

SUCH THAT INFINITY

EX AN EXPERIMENT INVOLVES FINDING THE PROPORTION OF INSECTS KILLED ON A PLANT AFTER AN INSECTICIDE IS APPLIED.

WHAT IS THE SAMPLE SPACE?

$S = \{x : 0 \le x \le 1\}$ OR 100%.

TYPES OF SAMPLE SPACES

DISCRETE CONSISTS OF A FINITE OR COUNTABLY INFINITE SET OF OUTCOMES.

$S = \{\text{HEADS}, \text{TAILS}\}$, $S = \{0, 1, 2, 3, \ldots\}$

CONTINUOUS CONSISTS OF INTERVALS

$S = \{x : 0 \le x \le 1\}$.

AN EVENT IS A SUBSET OF THE SAMPLE SPACE

2/4

EX Roll 2 dice and record result

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$

$(2,1), (2,2), (2,3),$

$\vdots$

$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

36 outcomes

A = Event that 1st is even.

$= \{(2,1), (2,2), \ldots, (2,6),$

$(4,1), (4,2), \ldots, (4,6),$

$(6,1), (6,2), \ldots, (6,6)\}.$

B = Event that sum is at least 10

$= \{(4,6), (5,5), (5,6), (6,4), (6,5), (6,6)\}$

Union of A and B is all elements in A or B.

$A \cup B = \{(2,1), (2,2), \ldots, (2,6),$

$(4,1), (4,2), \ldots, (4,6),$

$(6,1), (6,2), \ldots, (6,6),$

$(5,5), (5,6)\}$

Intersection of A and B is all elements in A and B.

$A \cap B = \{(4,6), (6,4), (6,5), (6,6)\}.$

If $A \cap B = \{\} = \phi$ then A and B are disjoint or mutually exclusive.

EX A: Event that 1st die is even.

C = Event that 1st die is odd

$A \cap C = \phi.$

3/4

<u>COMPLEMENT</u> OF AN EVENT A IS THE SET OF OUTCOMES IN S NOT IN A.

A', (ALSO $\bar{A}$, A^c)

<u>EX</u> A' = EVENT 1ST DIE IS ODD.

<u>EX</u> $S = \{0,1,2,3,4,5\}$

$A = \{0,1,2\}$

$B = \{2,3,4\}$

$C = \{5\}$

a) $A \cap B = \{2\}$

b) $A \cup C = \{0,1,2,5\}$

c) $(A \cup B)' = \{5\} = C$

d) $A \cup B \cup C = \{0,1,2,3,4,5\} = S$

NOTE: $A \cup A' = S$

$A \cap A' = \phi$

$(A')' = A$

<u>EX 2-28</u> EMISSIONS FROM 3 SUPPLIERS ARE CLASSIFIED FOR CONFORMANCE TO AIR QUALITY SPECIFICATIONS. THE RESULTS FROM 100 SAMPLES ARE:

A = EVENT THAT SAMPLE IS FROM SUPPLIER 1

B = EVENT THAT SAMPLE CONFORMS

	CONFORMS	
SUPPLIER	YES	NO
1	22	8
2	25	5
3	30	10

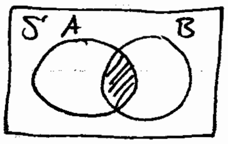
a) $A \cap B = 22$

b) $A' \cap B = 25 + 30 = 55$

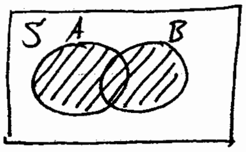
c) $B' = 8 + 5 + 10 = 23$

4/4

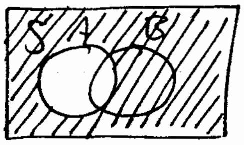
Venn Diagrams



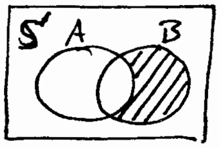
$A \cap B$



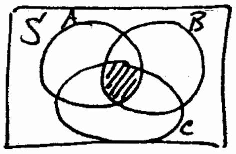
$A \cup B$



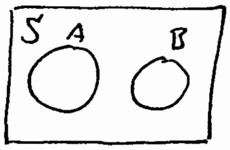
A'



$A' \cap B$



$A \cap B \cap C$



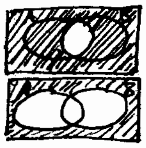
$A \cap B = \emptyset$, Disjoint Sets

Distributive Laws

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$
$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

DeMorgan's Laws

$$(A \cap B)' = A' \cup B'$$
$$(A \cup B)' = A' \cap B'$$



Ex A random experiment involves measuring the lifetime of an electrical component (in hours). Let A be the event that the component lasts no longer than 24 hours. Let B be the event that it lasts longer than 12 hours.

Find S, A, B, A', B', $A \cap B$.

$S = \{x : 0 \le x < \infty\}$ Continuous

$A = \{x : 0 \le x \le 24\}$ $A' = \{x : 24 < x < \infty\}$

$B = \{x : 12 < x < \infty\}$ $B' = \{x : 0 \le x \le 12\}$

$A \cap B = \{x : 12 < x \le 24\}$

2.2 ½ 2.2 PROBABILITY

EX $S = \{P_1, P_2, P_3, P_4\}$

RANDOM EXPERIMENT IS TO SELECT A PERSON AT RANDOM.

AT RANDOM = NO BIAS, EQUAL CHANCE OF SELECTION.

PROBABILITY THAT A PARTICULAR PERSON IS SELECTED IS $\frac{1}{4} = .25$.

PROBABILITY QUANTIFIES THE LIKELIHOOD OR CHANCE THAT AN OUTCOME OF A RANDOM EXPERIMENT OCCURS.

1. SUBJECTIVE PROBABILITY IS BASED ON DEGREE OF BELIEF.
2. RELATIVE FREQUENCY INTERPRETATION IS BASED ON REPEATED REPLICATIONS.

   THE PROBABILITY OF AN EVENT IS THE PROPORTION OF TIMES THE EVENT OCCURS IF AN EXPERIMENT IS REPEATED INDEFINATELY.

ASSIGNING PROBABILITY FOR DISCRETE SAMPLE SPACE.

- ASSUME EACH OF N OUTCOMES EQUALLY LIKELY.
- RANDOMLY SELECT 1 OBSERVATION OUT OF N
- PROBABILITY IS $1/N$
- PROBABILITY OF AN EVENT A IS THE SUM OF THE PROBABILITIES OF INDIVIDUAL OUTCOMES IN A.

EX ROLL A DIE AND RECORD RESULT.

$S = \{1, 2, 3, 4, 5, 6\}$ SO PROBABILITY OF ANY VALUE IS $1/6$.

$A = \{2, 4, 6\} \Rightarrow$ PROBABILITY OF A, $Pr(A) =$ ~~3/6 …~~

$= Pr(\text{ROLL } 2) + Pr(\text{ROLL } 4) + Pr(\text{ROLL}$

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2} = 0.$

$B = \{5, 6\} \Rightarrow Pr(B) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$.

FIND $Pr(B')$, $Pr(A \cap B)$, $Pr(A \cup B)$.

$B' = \{1, 2, 3, 4\} \Rightarrow Pr(B') = \frac{4}{6} = \frac{2}{3}$.

$Pr(A \cap B) = Pr(\text{ROLL } 6) = \frac{1}{6}$, $Pr(A \cup B) = Pr(\text{ROLL } 2, 4, 5, \text{OR } 6) = \frac{4}{6} = \frac{2}{3}$.

2/2

Axioms of Probability

Let E, E_1, and E_2 be events in S

1. $Pr(S) = 1$ (Something must occur)
2. $0 \le Pr(E) \le 1$
3. If $E_1 \cap E_2 = \emptyset$, then $Pr(E_1 \cup E_2) = Pr(E_1) + Pr(E_2)$

Ex Sampling with vs. without replacement (p. 20)

Suppose a batch consists of 4 parts $\{a, b, c, d\}$, and you sample 2 for inspection.

Let E be the event that part a gets picked.

Find $Pr(E)$ if the parts are selected

a) without replacement, and

b) with replacement.

a) Without Replacement.

$S = \{ab, ac, ad, ba, bc, bd, ca, cb, cd, da, db, dc\}$

12 outcomes

$E = \{ab, ac, ad, ba, ca, da\}$

$Pr(E) = \frac{6}{12} = \frac{1}{2} = 0.5$

b) With Replacement

$S = \{aa, ab, ac, ad, ba, bb, bc, bd, ca, cb, cc, cd, da, db, dc, dd\}$

16 outcomes

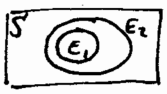
$E = \{aa, ab, ac, ad, ba, ca, da\}$

$Pr(E) = \frac{7}{16} = 0.4375$

Note: $Pr(\emptyset) = 0$

$Pr(E') = 1 - Pr(E)$

If $E_1 \underset{\text{subset}}{\subseteq} E_2$, then $Pr(E_1) \le Pr(E_2)$

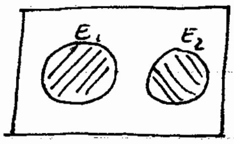


2.3 1/2

2.3 Addition Rules

Disjoint

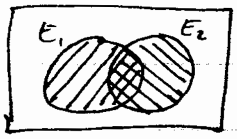
If $E_1 \cap E_2 = \phi$, then $Pr(E_1 \cup E_2) = Pr(E_1) + Pr(E_2)$



Not Disjoint

$Pr(E_1 \cup E_2) = Pr(E_1) + Pr(E_2) - Pr(E_1 \cap E_2)$

Subtract intersection so not counted twice.



Ex $S = \{1,2,3,4,5,6\}$

$A = \{2,4,6\}$, $B = \{5,6\}$.

Find $Pr(A \cup B)$.

$A \cup B = \{2,4,5,6\}$, $A \cap B = \{6\}$.

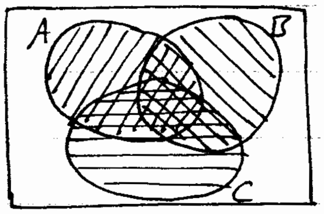
$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6}$.

$$Pr(A \cup B \cup C) = Pr(A) + Pr(B) + Pr(C) - Pr(A \cap B) - Pr(A \cap C) - Pr(B \cap C) + Pr(A \cap B \cap C).$$

Subtract 2-way intersections

Add back in 3-way intersection



Can consider as 2 events

$Pr(A \cup B \cup C) = Pr((A \cup B) \cup C)$

$= Pr((A \cup B)) + Pr(C) - Pr((A \cup B) \cap C)$

$= Pr(A) + Pr(B) - Pr(A \cap B) + Pr(C) - Pr((A \cup B) \cap C)$

$Pr((A \cup B) \cap C) = Pr((A \cap C) \cup (B \cap C))$

$= Pr((A \cap C)) + Pr((B \cap C)) - Pr((A \cap C) \cap (B \cap C))$

$Pr(A \cap B \cap C)$.

Substitute to get first result.

If a collection of events, $E_1, E_2, \ldots, E_K$, are <u>mutually exclusive</u>, that is,

$E_i \cap E_j = \phi$,

then $Pr(E_1 \cup E_2 \cup \cdots \cup E_K) = Pr(E_1) + Pr(E_2) + \cdots + Pr(E_K)$.

2/2 EX RECORDS AT AN INDUSTRIAL PLANT SHOW THAT

12% OF INJURED WORKERS ARE ADMITTED TO HOSPITAL,

16% " " " ARE BACK ON JOB THE NEXT DAY, AND

2% " " " ARE BOTH ADMITTED TO HOSPITAL AND BACK ON THE JOB THE NEXT DAY.

WHAT IS THE PROBABILITY AN INJURED WORKER IS EITHER ADMITTED TO HOSPITAL, BACK ON THE JOB THE NEXT DAY, OR BOTH?

A = EVENT ADMITTED TO HOSP $\quad P_r(A) = .12$

B = EVENT BACK TO WORK NEXT DAY $\quad P_r(B) = .16$

$P_r(A \cap B) = .02$.

$$P_r(A \cup B) = P_r(A) + P_r(B) - P_r(A \cap B)$$

$$= .12 + .16 - .02 = .26$$

2.4 ½

2.4 CONDITIONAL PROBABILITY

EX ROLL A DIE $S = \{1,2,3,4,5,6\}$

$A = \{2,4,6\}$, $B = \{6\}$, $A \cap B = \{6\}$

SUPPOSE WE KNOW AN EVEN NUMBER WAS ROLLED (EVENT A OCCURED), WHAT IS THE PROBABILITY THAT B OCCURS?

$Pr(B|A) = 1/3$

"GIVEN" OR "CONDITIONAL" ON A

$$Pr(B|A) = \frac{Pr(A \cap B)}{Pr(A)} = \frac{1/6}{1/2} = \frac{1}{6} \cdot \frac{2}{1} = \frac{2}{6} = \frac{1}{3}.$$

EX SAMPLES OF GALVANIZED STEEL ARE ANALYZED FOR COATING WEIGHT AND SURFACE ROUGHNESS.

A = EVENT COATING WT HIGH

B = EVENT SURFACE ROUGHNESS HIGH

SURFACE ROUGHNESS	COATING WT HIGH	COATING WT LOW	TOTAL
HIGH	12	16	28
LOW	88	34	122
TOTAL	100	50	150

a) FIND $Pr(B|A)$, IE PROB SURF HIGH GIVEN WT HIGH

b) FIND $Pr(A'|B)$, IE COATING LOW GIVEN SURF HIGH.

a) $$Pr(B|A) = \frac{Pr(A \cap B)}{Pr(A)} = \frac{12/150}{100/150} = \frac{12}{150} \cdot \frac{150}{100} = \frac{12}{100} = .12$$

b) $$Pr(A'|B) = \frac{Pr(A' \cap B)}{Pr(B)} = \frac{16/150}{28/150} = \frac{16}{28} = .57$$

EX SUPPOSE A BATCH CONTAINS 50 PARTS, 10 OF WHICH ARE DEFECTIVE.

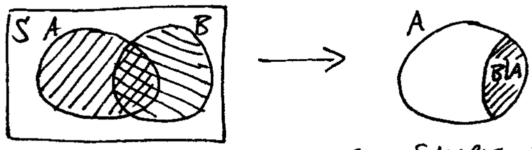
2 PARTS ARE SELECTED AT RANDOM WITHOUT REPLACEMENT.

WHAT IS THE PROBABILITY THAT SECOND PART IS NOT DEFECTIVE GIVEN THAT THE FIRST IS DEFECTIVE?

BY REASONING $Pr(2^{ND}\text{ NOT DEF.} \mid 1^{ST}\text{ DEF}) = \frac{40}{49}$ NON-DEFECTIVE LEFT AFTER 1^{ST} DEFECTIVE.

2/2

$$Pr(B|A) = \frac{Pr(A \cap B)}{Pr(A)}$$



NEW SAMPLE SPACE

EX 50 PARTS, 10 DEFECTIVE

BY FORMULA $Pr(2^{ND} \text{ NOT DEF} \mid 1^{ST} \text{ DEF}) = \dfrac{Pr(1^{ST} \text{ DEF AND } 2^{ND} \text{ NOT DEF})}{Pr(1^{ST} \text{ DEF})} = \dfrac{400/2450}{10/50} = \dfrac{4}{4}$

SELECTING 2 PARTS (w/o REPLACEMENT) FROM 50

10 DEF = $D_1, D_2, \ldots, D_{10}$

40 NOT : $N_1, N_2, \ldots, N_{40}$

$S = \{D_1D_2, D_1D_3, \ldots, D_1N_1, D_1N_2, \ldots, N_{39}N_{40}\}$

$50 \cdot 49 = 2450$ POSSIBLE ORDERED PAIRS.

NO REPLACEMENT

E = EVENT 1^{ST} DEF, 2^{ND} NOT DEF.

$10 \cdot 40 = 400$ OUTCOMES IN E.

$Pr(1^{ST} \text{ DEF AND } 2^{ND} \text{ NOT DEF}) = 400/2450$

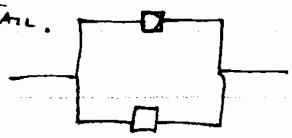
2.5 Multiplication and Total Probability Rules

Multiplication Rule

Algebraically rewrite conditional probability as a product

$$Pr(B|A) = \frac{Pr(A \cap B)}{Pr(A)} \Rightarrow Pr(A \cap B) = Pr(B|A)\,Pr(A)$$
$$= Pr(A|B)\,Pr(B)$$

Ex A two-component electrical system is connected in parallel, so that it fails only if both components fail.



The probability that first fails is .10.

If the first fails, the probability the second fails is .05.

What is the probability that the system fails?

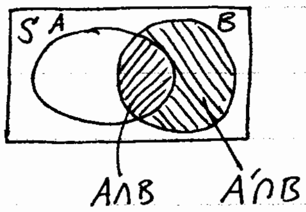
$$Pr(\text{system fails}) = Pr(\text{first fails and second fails})$$
$$= Pr(\text{second fails} \mid \text{first fails})\,Pr(\text{first fails})$$
$$= (.05)(.10) = .005.$$

Total Probability Rule

Consider finding $Pr(B)$

The set has two disjoint parts.

$A \cap B$ and $A' \cap B$.



$$Pr(B) = Pr(A \cap B) + Pr(A' \cap B)$$
$$= Pr(B|A)\,Pr(A) + Pr(B|A')\,Pr(A')$$

2/2

EX THE PROBABILITY IS 1% THAT AN ELECTRICAL CONNECTOR THAT IS KEPT DRY FAILS DURING THE WARRANTY PERIOD OF A PORTABLE COMPUTER. IF THE CONNECTOR IS EVER WET, THE PROBABILITY OF FAILURE DURING THE WARRANTY PERIOD IS 5%. IF 90% OF CONNECTORS ARE KEPT DRY, WHAT PROPORTION ARE EXPECTED TO FAIL DURING THE WARRANTY PERIOD? FIND $Pr(\text{FAIL})$.

$$Pr(\text{FAIL} \mid \text{DRY}) = .01 \qquad Pr(\text{DRY}) = .9$$

$$Pr(\text{FAIL} \mid \text{WET}) = .05 \qquad Pr(\text{WET}) = .1$$

$$Pr(\text{FAIL}) = Pr(\text{FAIL} \mid \text{DRY})\,Pr(\text{DRY}) + Pr(\text{FAIL} \mid \text{WET})\,Pr(\text{WET})$$

$$= (.01)(.9) + (.05)(.1)$$

$$= .014 = 1.4\%$$

2.6 1/2

2.6 INDEPENDENCE

WHAT HAPPENS IF KNOWING THAT A OCCURRED TELLS US NOTHING ABOUT B OCCURRING

EX A=FLIP COIN, B= ROLL DIE

$Pr(B|A) = Pr(B)$, A GIVES NO NEW INFO ABOUT B.

SAY A AND B ARE INDEPENDENT $(A \perp B)$

EX SELECT 2 PARTS OUT OF 50 WITH REPLACEMENT.

10 DEF, 40 NOT DEF.

WHAT IS THE PROBABILITY THAT SECOND IS NOT DEF GIVEN FIRST IS DEF?

$\cancel{\frac{40}{49}}$ $\frac{40}{50} = .8$ TWO EVENTS ARE INDEPENDENT.

TWO EVENTS ARE INDEPENDENT IF ANY ARE TRUE:

a) $Pr(A \cap B) = Pr(B|A)Pr(A)$
$= Pr(B)Pr(A)$

b) $Pr(B|A) = Pr(B)$

c) $Pr(A|B) = Pr(A)$

EX SUPPOSE EVENTS A AND B HAVE NON-ZERO PROBABILITIES AND ARE DISJOINT.

ARE A AND B INDEPENDENT?

CHECK: $Pr(A \cap B) \stackrel{?}{=} Pr(A)Pr(B)$

DISJOINT $\Rightarrow A \cap B = \phi$ SO $Pr(A \cap B) = 0 \neq Pr(A)Pr(B)$

BECAUSE THEY HAVE NON-ZERO PROBABILITIES.

SO A NOT INDEPENDENT OF B.

2/2 Notes:

- Independence is often assumed
- The only way to check independence is to do calculation
  Check (a), (b), or (c).
- Disjoint (or mutually exclusive) events are dependent.
- $Pr(A \cap B \cap C) = Pr(A) Pr(B) Pr(C)$ if A, B, and C are independent.

EX A production line is being monitored by a quality control engineer who discovers 20% of items being produced are defective.

a) If two items arrive in succession off the line, what is the probability both are defective?
   Assume conditions of the two items are independent.

b) If ten items arrive in succession, what is the probability that all are defective?

a) D_1 = Event item 1 is defective
   D_2 = Event item 2 is defective
   $Pr(D_1 \cap D_2)$ = Probability that both items 1 and 2 are defective
   By independence, $Pr(D_1 \cap D_2) = Pr(D_1) Pr(D_2) = (.2)(.2) = .04$.

b) $Pr(D_1 \cap D_2 \cap \cdots \cap D_{10})$ = Probability 10 items in a row are defective
   $= (.2)^{10} = 1.024 \times 10^{-7}$

2.7 1/1

2.7 BAYES' THEOREM

$$Pr(A \cap B) = Pr(A|B)\,Pr(B) = Pr(B|A)\,Pr(A)$$

$$Pr(A|B) = \frac{Pr(B|A)\,Pr(A)}{Pr(B)}, \quad Pr(B) > 0.$$

EX A TB TEST IS ADMINISTERED TO A RANDOMLY CHOSEN PERSON.

0.1 % OF POPULATION HAVE TB.

PROBABILITY TEST IS POSITIVE GIVEN THEY HAVE TB IS 0.98.

" " " " " " DO NOT HAVE TB IS 0.05

a) WHAT IS THE PROBABILITY OF TESTING POSITIVE?

b) IF THEY DO TEST POSITIVE, WHAT IS THE PROB. THEY HAVE TB?

LET E = EVENT THEY HAVE TB, + = POSITIVE TEST, − = NEGATIVE TEST

a) $Pr(+)$ GIVEN: $Pr(E) = .001$

b) $Pr(E|+)$ $Pr(+|E) = .98$

$Pr(+|E') = .05$

a) $Pr(+) = Pr(+|E)Pr(E) + Pr(+|E')Pr(E')$

$= (.98)(.001) + (.05)(1-.001)$

$= .05093$

b) $Pr(E|+) = \dfrac{Pr(E\cap +)}{Pr(+)} = \dfrac{Pr(+|E)Pr(E)}{Pr(+)}$

$= \dfrac{(.98)(.001)}{.05093} = 0.019242 \approx 2\%$

IS THIS AN EFFECTIVE TB TEST?

Quiz 3 Review (2.2-2.6)

Probability

If S is a discrete sample space with N equally likely outcomes, then the probability of each outcome is $1/N$.

For a discrete sample space, the probability of an event, $P(E)$, is the sum of the individual probabilities for the outcome in E.

Axioms of Probability

$Pr(S) = 1$

$0 \le Pr(E) \le 1$

If $A \cap B = \phi$, then $Pr(A \cup B) = Pr(A) + Pr(B)$

Properties

$Pr(E') = 1 - Pr(E)$

$Pr(\phi) = 0$

If $E_1 \subseteq E_2$, then $Pr(E_1) \le Pr(E_2)$

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

Conditional Probability

$$Pr(B|A) = \frac{Pr(A \cap B)}{Pr(A)}$$

Multiplication Rule

$$Pr(A \cap B) = Pr(B|A)\,Pr(A) = Pr(A|B)\,Pr(B)$$

Total Probability

$$Pr(B) = Pr(A \cap B) + Pr(A' \cap B)$$
$$= Pr(B|A)\,Pr(A) + Pr(B|A')\,Pr(A')$$

A, B are independent if any of these are true

$Pr(A \cap B) = Pr(A)\,Pr(B)$

$Pr(A|B) = Pr(A)$

$Pr(B|A) = Pr(B)$

Ex Samples of emissions from three plants are classified for compliance of air quality standards.

A = Event from Plant 1

B = Event conforms

Plant	Conforms Y	N
1	22	8
2	25	5
3	30	10

a) Find $P(A|B)$, b) Are A, B indep?

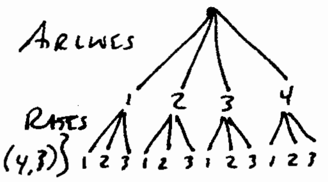
a) $Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{22/100}{77/100} = \frac{22}{77} = \frac{2}{7} = .286$

b) $Pr(A|B) \stackrel{?}{=} Pr(A)$

$\frac{2}{7} \stackrel{?}{\neq} \frac{30}{100} = \frac{3}{10}$, not indep.

2.1 CD Counting Techniques (For Sample Spaces and Simple Events)

EX A Product can be shipped by 4 different airlines and each can be shipped by 3 different rates.
How many distinct ways can product be shipped?



Sample Space $S = \{(1,1), (1,2), (1,3), (2,1), \ldots, (4,3)\}$ → 12 ways

Multiplication Rule

If you have k sets with n_1 in 1st set, n_2 in 2nd set, …, n_k in k^{th} set, you can form a sample of k elements by taking 1 from each set.
This gives $n_1 \cdot n_2 \cdots n_k$ different samples.

EX Toss a coin 4 times and roll a die 3 times.

$\underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{6}\ \underline{6}\ \underline{6} = 3456$ possible samples.

EX There are 5 different space flights, each requiring 1 astronaut; no astronaut can go on more than one flight. How many ways can 5 astronauts be selected from the 100 candidates?

$\underline{100}\ \underline{99}\ \underline{98}\ \underline{97}\ \underline{96}$ (Sample 5 without replacement)

$= 9{,}034{,}502{,}400$

$= \dfrac{100!}{95!} = \dfrac{100!}{(100-5)!} = \dfrac{n!}{(n-k)!}$ Note: $5! = 5\cdot4\cdot3\cdot2\cdot1$

Permutation Rule

The number of permutations (orderings) of a subset of k elements selected from a set of n different elements is

$$P^n_k = \frac{n!}{(n-k)!} = n(n-1)(n-2)\cdots(n-k+1)$$

2/2 EX HAVE 4 CDS AND WE SELECT 3 TO PLAY

HOW MANY WAYS CAN WE DO THIS IF

a) ORDER IS IMPORTANT?

b) ORDER IS NOT IMPORTANT?

a) 4 CDS A, B, C, D, NUMBER OF ORDERINGS IS $P^4_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 4 \cdot 3 \cdot 2 = 24$

A B C	B A C	C A B	D A B
A B D	B A D	C A D	D A C
A C B	B C A	C B A	D B A
A C D	B C D	C B D	D B C
A D B	B D A	C D A	D C A
A D C	B D C	C D B	D C B

b) REMOVE REPEATS, ONLY 4 LEFT

$$\frac{24}{6} = 4 = \frac{24}{3 \cdot 2 \cdot 1}$$

COMBINATION RULE

THE NUMBER OF SUBSETS OF SIZE k THAT CAN BE SELECTED FROM A SET OF n ELEMENTS IS

$$\binom{n}{k} = C^n_k = \frac{n!}{k!(n-k)!} \qquad \text{"}n \text{ CHOOSE } k\text{"}$$

$\binom{n}{k}$ IMPORTANT TO KNOW AND UNDERSTAND.