

Stat 345 Solutions - Section 6.5 (2nd edition)

Problem 6-63

First, we find the marginal distributions. The marginal distribution for X is

x	1	2	3
$f_X(x)$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

The marginal distribution for Y is

y	3	4	5	6
$f_Y(y)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$

The covariance is given by

$$\text{cov}(X, Y) = E(XY) - E(X)E(Y)$$

First, we find $E(XY)$:

$$\begin{aligned} E(XY) &= \sum_x \sum_y xy f_{XY}(x, y) \\ &= (1)(3)\frac{1}{8} + (1)(4)\frac{1}{4} + (2)(5)\frac{1}{2} + (3)(6)\frac{1}{8} \\ &= 8.625 \end{aligned}$$

Now we find $E(X)$ and $E(Y)$:

$$E(X) = (1)\frac{3}{8} + (2)\frac{1}{2} + (3)\frac{1}{8} = 1.75$$

$$E(Y) = (3)\frac{1}{8} + (4)\frac{1}{4} + (5)\frac{1}{2} + (6)\frac{1}{8} = 4.625$$

$$\text{Thus, } \text{cov}(X, Y) = 8.625 - (1.75)(4.625) = 0.5313.$$

The correlation is given by

$$\text{corr}(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}Y}}$$

So we need to find the variances:

$$\text{Var}(X) = E(X^2) - E(X)^2 = (1)\frac{3}{8} + (4)\frac{1}{2} + (9)\frac{1}{8} - 1.75^2 = 0.4375$$

$$\text{Var}(Y) = E(Y^2) - E(Y)^2 = (9)\frac{1}{8} + (16)\frac{1}{4} + (25)\frac{1}{2} + (36)\frac{1}{8} - 4.625^2 = 0.7344$$

Thus the correlation is

$$\text{corr}(X, Y) = \frac{0.5313}{\sqrt{0.43750.7344}} = 0.9373.$$