

Stat 345 Solutions - Section 5.5 (3rd edition)

Problem 5-67

First, we find the marginal distributions. The marginal distribution for X is

x		1	2	4
$f_X(x)$		$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

The marginal distribution for Y is

y		3	4	5	6
$f_Y(y)$		$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$

The covariance is given by

$$\text{cov}(X, Y) = E(XY) - E(X)E(Y)$$

First, we find $E(XY)$:

$$\begin{aligned} E(XY) &= \sum_x \sum_y xy f_{XY}(x, y) \\ &= (1)(3)\frac{1}{8} + (1)(4)\frac{1}{4} + (2)(5)\frac{1}{2} + (4)(6)\frac{1}{8} \\ &= 9.375 \end{aligned}$$

Now we find $E(X)$ and $E(Y)$:

$$E(X) = (1)\frac{3}{8} + (2)\frac{1}{2} + (4)\frac{1}{8} = 1.875$$

$$E(Y) = (3)\frac{1}{8} + (4)\frac{1}{4} + (5)\frac{1}{2} + (6)\frac{1}{8} = 4.625$$

$$\text{Thus, } \text{cov}(X, Y) = 9.375 - (1.875)(4.625) = 0.7031.$$

The correlation is given by

$$\text{corr}(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{\text{Var}(x)\text{Var}Y}}$$

So we need to find the variances:

$$\text{Var}(X) = E(X^2) - E(X)^2 = (1)\frac{3}{8} + (4)\frac{1}{2} + (16)\frac{1}{8} - 1.875^2 = 0.8594$$

$$\text{Var}(Y) = E(Y^2) - E(Y)^2 = (9)\frac{1}{8} + (16)\frac{1}{4} + (25)\frac{1}{2} + (36)\frac{1}{8} - 4.625^2 = 0.7344$$

Thus the correlation is

$$\text{corr}(X, Y) = \frac{0.7031}{\sqrt{0.85940.7344}} = 0.8850$$