

Stat 345 Solutions - Section 4.8 (2nd ed.)/3.8 (3rd ed.)

Problem 4-74/3-86

The pmf of X is

$$f(x) = \frac{\binom{20}{x} \binom{80}{4-x}}{\binom{100}{4}}, x=0, 1, 2, 3, 4$$

(a)

$$\begin{aligned} P(X = 1) &= \frac{\binom{20}{1} \binom{80}{4-1}}{\binom{100}{4}} \\ &= 0.4191 \end{aligned}$$

(b) $P(X = 6) = 0$

(c)

$$\begin{aligned} P(X = 4) &= \frac{\binom{20}{4} \binom{80}{4-4}}{\binom{100}{4}} \\ &= 0.0012 \end{aligned}$$

(d) $\mu = n\left(\frac{K}{N}\right) = 4(20/100) = 0.8$

$$\sigma^2 = n\left(\frac{K}{N}\right)\left(1 - \frac{K}{N}\right)\left(\frac{N-n}{N-1}\right) = 4(20/100)\left(1 - 20/100\right)\left(\frac{100-4}{100-1}\right) = 0.6206$$

Problem 4-76/3-88

The pmf of X is

$$f(x) = \frac{\binom{4}{x} \binom{6}{3-x}}{\binom{10}{3}}, x=0, 1, 2, 3$$

Using this expression, the probabilities assigned to each outcome are:

$$P(X = 0) = 0.167$$

$$P(X = 1) = 0.5$$

$$P(X = 2) = 0.3$$

$$P(X = 3) = 0.033$$

The graph would have the numbers 0 - 3 on the x-axis, and 0-1 on the y-axis. Each of the points 0, 1, 2, 3, would have a spike whose height is given by the probabilities above.

Problem 4-77/3-89

The cdf is

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.167 & 0 \leq x < 1 \\ 0.667 & 1 \leq x < 2 \\ 0.967 & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

Problem 4-78/3-90

Let the random variable X be the number of unacceptable washers in the sample. Then X has a hypergeometric distribution with $N = 75$, $n = 10$, and $K = 5$.

The pmf of X is

$$f(x) = \frac{\binom{5}{x} \binom{70}{10-x}}{\binom{75}{10}}, \quad x=0, 1, 2, 3, 4, 5$$

(a)

$$\begin{aligned} P(X = 0) &= \frac{\binom{5}{0} \binom{70}{10-0}}{\binom{75}{10}} \\ &= 0.4786 \end{aligned}$$

(b)

$$\begin{aligned} P(X \geq 1) &= 1 - P(X < 1) \\ &= 1 - P(X = 0) \\ &= 1 - 0.4786 \\ &= 0.5214 \end{aligned}$$

(c)

$$\begin{aligned} P(X = 1) &= \frac{\binom{5}{1} \binom{70}{10-1}}{\binom{75}{10}} \\ &= 0.3923 \end{aligned}$$

(d) $\mu = 10(5/75) = 0.667$