STAT 345 - Distributions Handout
BASED ON SECTIONS: 3.5 – 3.9, 4.5 – 4.7, 4.9

Discrete Distributions

3.5 Discrete Uniform Distribution

Description X has a descrete uniform distribution if the probability of each of the N values in its range is equally likely.

Notation $X \sim \text{DiscreteUniform}(\frac{1}{N})$,
$X =$ value in range,
$N =$ number of values

PMF $\Pr(X = x) = f(x) = \frac{1}{N}, \quad x = 1, 2, \ldots, N$

Mean $\mu = \text{E}(X) = \frac{N+1}{2}$

Variance $\sigma^2 = \text{Var}(X) = \frac{N^2-1}{12} = \frac{(N+1)(N-1)}{12}$

3.6 Bernoulli (trials) Distribution

Description X has a Bernoulli distribution if it results in one of two possible outcomes, labelled “success” and “failure”.

Notation $X \sim \text{Bern}(p)$,
$X =$ number of “successes”,
$p =$ probability of success

PMF $\Pr(X = x) = f(x) = p^x(1-p)^{1-x}, \quad x = 0, 1,$

Mean $\mu = \text{E}(X) = p$

Variance $\sigma^2 = \text{Var}(X) = p(1-p)$

3.6 Binomial Distribution

Description X has a binomial distribution if it counts the number of successes in n independent trials (items) with common probability of success. Also, X is binomial if it is the sum of Bernoulli trials.

Notation $X \sim \text{Bin}(n, p)$,
$X =$ number of “successes”,
$n =$ number of trials,
$p =$ probability of success

PMF $\Pr(X = x) = f(x) = \binom{n}{x} p^x(1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$

Mean $\mu = \text{E}(X) = np$

Variance $\sigma^2 = \text{Var}(X) = np(1-p)$

Note The Bernoulli distribution is the Binomial with $n = 1$.

3.7 Geometric Distribution

Description X has a geometric distribution if it counts the number of Bernoulli trials until the *first* “success”.

Notation $X \sim \text{Geom}(p)$,
$X =$ number of trials until first “success”,
$p =$ probability of success

PMF $\Pr(X = x) = f(x) = (1-p)^{x-1}p^1, \quad x = 1, 2, \dots$

Mean $\mu = \text{E}(X) = \frac{1}{p}$

Variance $\sigma^2 = \text{Var}(X) = \frac{1-p}{p^2}$

3.7 Negative Binomial Distribution

Description X has a negative binomial distribution if it counts the number of Bernoulli trials until the r^{th} “success”.

Notation $X \sim \text{NegBin}(r, p)$,
$X =$ number of trials until r^{th} “success”,
$r =$ number of successes,
$p =$ probability of success

PMF $\Pr(X = x) = f(x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r, \quad x = r, r+1, r+2, \dots$

Mean $\mu = \text{E}(X) = \frac{r}{p}$

Variance $\sigma^2 = \text{Var}(X) = \frac{r(1-p)}{p^2}$

Note The Geometric distribution is the Negative Binomial with $r = 1$.

3.8 Hypergeometric Distribution

Description X has a hypergeometric distribution if it counts the number of “successes” when drawing n objects without replacement from a set of N objects where K are labelled as “successes” and $N - K$ are labelled as “failures”.

Notation $X \sim \text{HyperGeom}(N, K, n)$,
$X =$ number of “successes”,
$N =$ population size (total number of objects),
$K =$ number of successes in population,
$n =$ number of objects drawn

PMF $\Pr(X = x) = f(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}, \quad x = 0, 1, 2, \dots, K$

Mean $\mu = \text{E}(X) = n\frac{K}{N}, np$, where $p = \frac{K}{N}$ is the probability of success

Variance $\sigma^2 = \text{Var}(X) = n\frac{K}{N}\frac{N-K}{N}\frac{N-n}{N-1} = np(1-p)\frac{N-n}{N-1}$, where $\frac{N-n}{N-1}$ is the finite population correction

Note The Hypergeometric distribution is the Binomial with a known finite population size, N.

3.9 Poisson Distribution

Description X has a Poisson distribution if it counts events in an interval, often "rare events" ($p \leq 0.1$).

Notation $X \sim \text{Pois}(\lambda)$,
$X =$ number of events,
$\lambda =$ average number of events in interval

PMF $\Pr(X = x) = f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$

Mean $\mu = \text{E}(X) = \lambda$

Variance $\sigma^2 = \text{Var}(X) = \lambda$

Continuous Distributions

4.5 Continuous Uniform Distribution

Description X has a continuous uniform distribution if every value in an interval has the same likelihood.

Notation $X \sim \text{U}(a, b)$,
$X =$ value from interval,
$a =$ left interval endpoint,
$b =$ right interval endpoint

PDF $\Pr(X = x) = f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$

CDF $\Pr(X \leq x) = F(x) = \frac{x-a}{b-a}, \quad a \leq x \leq b$

Mean $\mu = \text{E}(X) = \frac{a+b}{2}$

Variance $\sigma^2 = \text{Var}(X) = \frac{(b-a)^2}{12}$

4.9 Exponential Distribution

Description X has an exponential distribution if it measures the distance between successive counts in a Poisson process.

Notation $X \sim \text{Exp}(\lambda)$,
$X =$ distance until event,
$\lambda =$ average number of events per unit of Poisson process

PDF $\Pr(X = x) = f(x) = \lambda e^{-\lambda x}, \quad 0 \leq x < \infty, \lambda > 0$

CDF $\Pr(X \leq x) = F(x) = 1 - e^{-\lambda x}, \quad 0 \leq x < \infty$

Mean $\mu = \text{E}(X) = \frac{1}{\lambda}$

Variance $\sigma^2 = \text{Var}(X) = \frac{1}{\lambda^2}$

4.6 Normal Distribution

Description X has a normal distribution if it measures a process that has a center and is subject to error or symmetric deviations, for example. It is also the distribution of means.

Notation $X \sim \mathrm{N}(\mu, \sigma^2)$,
$X =$ value from distribution,
$\mu =$ center of distribution,
$\sigma^2 =$ spread of distribution

PDF $\Pr(X = x) = f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{1}{2\sigma^2}(X-\mu)^2}$,
$-\infty < x < \infty, -\infty < \mu < \infty, \sigma^2 > 0$

CDF $\Pr(Z \leq z) = F(z) = \Phi(z) =$ look up in Table II, pp.653–4.

Mean $\mu = \mathrm{E}(X) = \mu$

Variance $\sigma^2 = \mathrm{Var}(X) = \sigma^2$

Note $Z = \frac{X-\mu}{\sigma} \sim \mathrm{N}(0,1)$ is the Standard Normal distribution, the quantiles of which are tabulated in Table II.

4.7 Normal approximation to the Binomial and Poisson Distributions

Binomial If both $np > 5$ and $n(1-p) > 5$,
then take $\mu = np$ and $\sigma^2 = np(1-p)$ so that
$Z = \frac{X-\mu}{\sigma} = \frac{X-np}{\sqrt{np(1-p)}} \overset{approx}{\sim} N(0,1)$.

Poisson If $\lambda > 5$,
then take $\mu = \lambda$ and $\sigma^2 = \lambda$ so that
$Z = \frac{X-\mu}{\sigma} = \frac{X-\lambda}{\sqrt{\lambda}} \overset{approx}{\sim} N(0,1)$.